Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 5 [Probability & Statistics 1]

Exam Series: May 2015 – May 2022

Format Type B:
Each question is followed by its answer scheme

Chapter 3

Probability

132. 9709_s22_qp_51 Q: 6

Janice is playing a computer game. She has to complete level 1 and level 2 to finish the game. She is allowed at most two attempts at any level.

- For level 1, the probability that Janice completes it at the first attempt is 0.6. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.3.
- If Janice completes level 1, she immediately moves on to level 2.
- For level 2, the probability that Janice completes it at the first attempt is 0.4. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.2.

(a) Show that the probability that Janice moves on to level 2 is 0.72. [1]

(b) Find the probability that Janice finishes the game. [3]

(c) Find the probability that Janice fails exactly one attempt, given that she finishes the game. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$0.6+0.4\times0.3=0.72$ or $1-0.4\times0.7=0.72$	B1	Clear identified calculation AG
		1	
(b)	$0.72\times(0.4+0.6\times0.2)$	M1	$0.72\times u$, $0<u<1$
		M1	$v\times(0.4+0.6\times0.2)$, or $v\times(1-0.6\times0.8)$ $0<v\leqslant1$ no additional terms **SC B1** for $0.72\times(0.4+0.12)$ or $0.72\times(1-0.48)$
	0.3744	A1	WWW. Condone 0.374. **SC B1** for 0.3744 only
		3	
	Alternative method for question 6(b)		
	[p(P1P2) + p(F1P1P2) + p(P1F2P2) + p(F1P1F2P2)] = $0.6\times0.4+0.4\times0.3\times0.4+0.6\times0.6\times0.2+0.4\times0.3\times0.6\times0.2$	M1	Any two terms unsimplified and correct
		M1	Summing 4 appropriate scenarios by listing or on a tree diagram **SC B1** for $0.24+0.048+0.072+0.0144$
	0.3744	A1	WWW. Condone 0.374. **SC B1** for 0.3744 only
		3	

Question	Answer	Marks	Guidance
(c)	$P(\text{fails first or second level}\mid\text{finishes game})=\dfrac{P(\text{fails first or second level}\cap\text{finishes game})}{their\,(\mathbf{b})}$	M1	Either $0.6\times0.6\times0.2$ or $0.4\times0.3\times0.4$ seen Condone 0.072 or 0.048 if seen in **(b)**
	Numerator = P(S SF) + P(FS S) = $0.6\times0.6\times0.2+0.4\times0.3\times0.4=0.072+0.048=0.12$	A1	Both correct accept unsimplified expression. No additional terms
	Required probability = $\dfrac{0.12}{their\,(\mathbf{b})}$	M1	$\dfrac{\textit{Their}\text{ sum of two 3-term probabilities as numerator}}{\textit{their}\ (\mathbf{b})\text{ or correct}}$
	0.321 or $\dfrac{25}{78}$	A1	$0.3205<p\leqslant0.321$
		4	

133. 9709_s22_qp_52 Q: 7

Hanna buys 12 hollow chocolate eggs that each contain a sweet. The eggs look identical but Hanna knows that 3 contain a red sweet, 4 contain an orange sweet and 5 contain a yellow sweet. Each of Hanna's three children in turn randomly chooses and eats one of the eggs, keeping the sweet it contained.

(a) Find the probability that all 3 eggs chosen contain the same colour sweet. [4]

(b) Find the probability that all 3 eggs chosen contain a yellow sweet, given that all three children have the same colour sweet. [2]

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(c) Find the probability that at least one of Hanna's three children chooses an egg that contains an orange sweet. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	YYY: $\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10} = \frac{60}{1320},\frac{1}{22}$ OOO: $\frac{4}{12}\times\frac{3}{11}\times\frac{2}{10} = \frac{24}{1320},\frac{1}{55}$ RRR: $\frac{3}{12}\times\frac{2}{11}\times\frac{1}{10} = \frac{6}{1320},\frac{1}{220}$	M1	**Either** $12 \times 11 \times 10$ in denominator **or** $a \times (a-1) \times (a-2)$, $a = 5, 4, 3$ in numerator seen in at least one expression.
		A1	One expression $\frac{a}{12}\times\frac{a-1}{11}\times\frac{a-2}{10}$, $a = 5, 4, 3$ (consistent in expression). Correct order of values in the numerator is essential.
		M1	$\frac{5}{12}\times\frac{4}{d}\times\frac{3}{e}+\frac{4}{12}\times\frac{3}{d}\times\frac{2}{e}+\frac{3}{12}\times\frac{2}{d}\times\frac{1}{e}$, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$. Condone $\frac{1}{22}+\frac{1}{55}+\frac{1}{220}$ OE
	[Total =] $\frac{90}{1320},\frac{3}{44}$, 0.0682	A1	0.06818. Dependent only upon the second M mark.

Question	Answer	Marks	Guidance
(a)	**Alternative method for question 7(a)**		
	YYY: $\frac{^5C_3}{^{12}C_3} = \frac{10}{220},\frac{1}{22}$ OOO: $\frac{^4C_3}{^{12}C_3} = \frac{4}{220},\frac{1}{55}$ RRR: $\frac{^3C_3}{^{12}C_3} = \frac{1}{220}$	M1	Either $^{12}C_3$ in denominator or $^aC_3$ in numerator seen in at least one expression.
		A1	One expression $\frac{^aC_3}{^{12}C_3}$ $a = 5, 4, 3$
		M1	$\frac{^5C_3}{^{12}C_3}+\frac{^4C_3}{^{12}C_3}+\frac{^3C_3}{^{12}C_3}$ Condone $\frac{1}{22}+\frac{1}{55}+\frac{1}{220}$ OE
	[Total =] $\frac{90}{1320},\frac{3}{44}$, 0.0682	A1	0.06818. Dependent only upon the second M mark.
		4	
(b)	[P(YYY | all same colour) =] $\frac{60}{1320}\div\frac{90}{1320}$	M1	$\dfrac{\textit{their}\ P(YYY) \text{ or } \frac{60}{1320} \text{ or } \frac{1}{22}}{\textit{their}\ 7(a) \text{ or } \frac{90}{1320} \text{ or } \frac{3}{44}}$
	$\frac{2}{3}$, 0.667	A1	OE
		2	

Question	Answer	Marks	Guidance
(c)	In each method, the M mark requires the scenarios to be identifiable. This may be implied by a list of scenarios and then the calculations which will be assumed to be in the same order. A correct value/expression will be condoned as identifying the connected scenario.		
	Method 1		
	[1 – no orange =] $1-\frac{8}{12}\times\frac{7}{11}\times\frac{6}{10}$ or $1-\frac{^8C_3}{^{12}C_3} = 1-\frac{14}{55}$	B1	$\frac{8}{12}\times\frac{7}{11}\times\frac{6}{10}$ or $\frac{^8C_3}{^{12}C_3}$ seen, condone $\frac{336}{1320}$ or $\frac{56}{220}$ only, not OE.
		M1	$1-\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ Either $d = 11$, $e = 10$ or $d = 12$, $e = 12$ or $1 - \frac{^8C_3}{^{12}C_3}$. Condone $1-\frac{14}{55}$ OE (not $\frac{41}{55}$).
	$\frac{41}{55}$	A1	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

Question	Answer	Marks	Guidance
(c)	**Method 2**		
	$P(1\ O)=\left(\frac{4}{12}\times\frac{3}{11}\times\frac{2}{10}+\frac{4}{12}\times\frac{5}{11}\times\frac{4}{10}+2\times\frac{4}{12}\times\frac{5}{11}\times\frac{3}{10}\right)\times 3=\frac{672}{1320}$ $P(2O)=\frac{4}{12}\times\frac{3}{11}\times\frac{8}{10}\times 3=\frac{288}{1320}$ $P(3O)=\frac{24}{1320}$	**B1**	P(1 O)or P(2 O) correct, accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$.
	[Total =] $\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.
	Method 3		
	$O\ Y\ R = {}^4C_1\times{}^5C_1\times{}^3C_1 = 60$ $O\ R\ R = {}^4C_1\times{}^3C_2 = 12$ $O\ Y\ Y = {}^4C_1\times{}^5C_2 = 40$ $O\ O\ Y = {}^4C_2\times{}^5C_1 = 30$ $O\ O\ R = {}^4C_2\times{}^3C_1 = 18$ $O\ O\ O = {}^4C_3 = 4$ Total = 164 Prob $=\frac{164}{{}^{12}C_3}$	**B1**	Number of ways either 1 or 2 orange sweets obtained correctly (112 or 48). Accept unsimplified Note ${}^4C_1\times{}^8C_2 = 112$ or ${}^4C_2\times{}^8C_1 = 48$ are correct alternatives.
		M1	3 correct scenarios (1, 2 or 3 orange sweets) added on numerator, denominator ${}^{12}C_3$
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

Question	Answer	Marks	Guidance
(c)	**Method 4**		
	$P(R\ R\ O)=\frac{3}{12}\times\frac{2}{11}\times\frac{4}{10}=\frac{1}{55}$ $P(R\ O\ \ )=\frac{3}{12}\times\frac{4}{11}=\frac{1}{11}$ $P(R\ Y\ O)=\frac{3}{12}\times\frac{5}{11}\times\frac{4}{10}=\frac{1}{22}$ $P(O\ \ \ )=\frac{4}{12}=\frac{1}{3}$ $P(Y\ R\ O)=\frac{5}{12}\times\frac{3}{11}\times\frac{4}{10}=\frac{1}{22}$ $P(Y\ O\ \ )=\frac{5}{12}\times\frac{4}{11}=\frac{5}{33}$ $P(Y\ Y\ O)=\frac{5}{12}\times\frac{4}{11}\times\frac{4}{10}=\frac{2}{33}$	**B1**	$P(R\ \hat{}\ \hat{}) = \frac{17}{110}$ or $P(Y\ \hat{}\ \hat{}) = \frac{17}{66}$. Accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$.
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

Question	Answer	Marks	Guidance
(c)	**Method 5**		
	$P(O\ \ \ )=\frac{4}{12}=\frac{1}{3}$ $P(\hat{}\ O\ \ )=\frac{8}{12}\times\frac{4}{11}=\frac{8}{33}$ $P(\hat{}\ \hat{}\ O)=\frac{8}{12}\times\frac{7}{11}\times\frac{4}{10}=\frac{28}{165}$	**B1**	$P(\hat{}\ O\ \ ) = \frac{8}{33}$ or $P(\hat{}\ \hat{}\ O) = \frac{28}{165}$. Accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$ with correct numerator.
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.
		3	

134. 9709_s22_qp_53 Q: 6

Sajid is practising for a long jump competition. He counts any jump that is longer than 6 m as a success. On any day, the probability that he has a success with his first jump is 0.2. For any subsequent jump, the probability of a success is 0.3 if the previous jump was a success and 0.1 otherwise. Sajid makes three jumps.

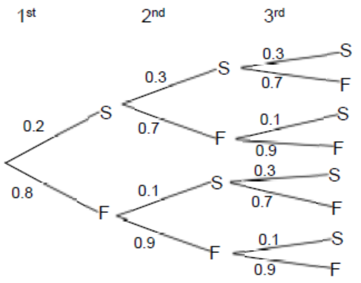
(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]

(b) Find the probability that Sajid has exactly one success given that he has at least one success. [5]

On another day, Sajid makes six jumps.

(c) Find the probability that only his first three jumps are successes or only his last three jumps are successes. [3]

Answer:

Question	Answer	Marks	Guidance
(a)		B1	First and second jumps correct with probabilities and outcomes identified.
		B1	Third jump correct with probabilities and outcomes identified.
		2	
(b)	SFF $0.2\times0.7\times0.9 = 0.126$ FSF $0.8\times0.1\times0.7 = 0.056$ FFS $0.8\times0.9\times0.1 = 0.072$	M1	Two or three correct 3 factor probabilities added, correct or FT from part **6(a)**. Accept unsimplified.
	[Total = probability of 1 success =] 0.254 $\left(\frac{127}{500}\right)$	A1	Accept unsimplified.
	[Probability of at least 1 success = $1-0.8\times0.9\times0.9$ =]0.352 $\left(\frac{44}{125}\right)$	B1 FT	Accept unsimplified.
	P(exactly 1 success | at least 1 success)= $\frac{\textit{their}\ 0.254}{\textit{their}\ 0.352}$	M1	Accept unsimplified.
	0.722, $\frac{127}{176}$	A1	$0.7215 < p \leqslant 0.722$
		5	

Question	Answer	Marks	Guidance
(c)	$0.8\times0.9\times0.9\times0.1\times0.3\times0.3 = 0.005832$ [FFFSSS] $0.2\times0.3\times0.3\times0.7\times0.9\times0.9 = 0.010206$ [SSSFFF]	M1	$a\times b\times c\times d\times e\times f$ FT from *their* tree diagram. Either *a*, *b* and *c* all = 0.8 or 0.9 (at least one of each) and *d*, *e* and *f* all = 0.1 or 0.3 (at least one of each). Or *a*, *b*, *c* = 0.2 or 0.3 (at least one of each) and *d*, *e*, *f* = 0.7 or 0.9 (at least one of each).
		A1	Either correct. Accept unsimplified.
	[Total =] 0.0160[38]	A1	
		3	

135. 9709_m21_qp_52 Q: 2

Georgie has a red scarf, a blue scarf and a yellow scarf. Each day she wears exactly one of these scarves. The probabilities for the three colours are 0.2, 0.45 and 0.35 respectively. When she wears a red scarf, she always wears a hat. When she wears a blue scarf, she wears a hat with probability 0.4. When she wears a yellow scarf, she wears a hat with probability 0.3.

(a) Find the probability that on a randomly chosen day Georgie wears a hat. [2]

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(b) Find the probability that on a randomly chosen day Georgie wears a yellow scarf given that she does not wear a hat. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	$0.2[\times 1]+0.45\times 0.4+0.35\times 0.3$	M1	$0{\cdot}2\ [\times\ 1] + 0{\cdot}45 \times b + 0{\cdot}35 \times c$, $b = 0{\cdot}4$, $0{\cdot}6$ $c = 0{\cdot}3$, $0{\cdot}7$
	$0{\cdot}485$ or $\frac{97}{200}$	A1	
		2	
(b)	$P(Y\mid\overline{H})=\frac{P(Y\cap\overline{H})}{P(\overline{H})}=\frac{0.35\times 0.7}{1-\mathit{their}\,\mathbf{(a)}}=\frac{0.245}{0.515}$	B1	$0{\cdot}35 \times 0{\cdot}7$ or $0{\cdot}245$ seen as numerator or denominator of fraction.
		M1	$0{\cdot}515$ or 1 – *their* **(a)** or $[0{\cdot}3 \times 0 +]\ 0{\cdot}45 \times d + 0{\cdot}35 \times e$, where d = *their* b', e = *their* c' seen as denominator of fraction.
	$0{\cdot}476$ or $\frac{49}{103}$	A1	$0{\cdot}4757 \leqslant p \leqslant 0{\cdot}476$
		3	

136. 9709_s21_qp_51 Q: 3

(a) How many different arrangements are there of the 8 letters in the word RELEASED? [1]

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(b) How many different arrangements are there of the 8 letters in the word RELEASED in which the letters LED appear together in that order? [3]

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(c) An arrangement of the 8 letters in the word RELEASED is chosen at random.

Find the probability that the letters A and D are not together. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$\left[\frac{8!}{3!}\right] = 6720$	B1	NFWW, must be evaluated
		1	
(b)	_ _ _ L E D _ _ : With LED together: $\frac{6!}{2!}$	M1	$\frac{6!}{k}$ or $\frac{5!x6}{k}$ $k \geqslant 1$ and no other terms
		M1	$\frac{m}{2!}$, m an integer, $m \geqslant 5$
	360	A1	CAO
		3	
(c)	Method using _ _ _ A _ D _ _ : Arrange the 6 letters RELESE = $\frac{6!}{3!}$ [= 120]	*M1	$\frac{6!}{3!} \times k$ seen, k an integer > 0
	Multiply by number of ways of placing AD in non-adjacent places = *their* 120 × $^7P_2$ [= 5040]	*M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n = 6, 7 or 8, m an integer > 0
	[Probability =] $\frac{\textit{their } 5040}{\textit{their } 6720}$	DM1	Denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using 'Total arrangements – Arrangements with A and D together': *Their* 6720 – $\frac{7! \times 2}{3!}$ [= 5040]	*M1	*Their* 6720 – k, k a positive integer
		*M1	$(m-)\frac{7! \times k}{3!}, k = 1, 2$
	[Probability =] $\frac{\textit{their } 5040}{\textit{their } 6720}$	DM1	With denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using '1 – Probability of arrangements with A and D together': $\frac{7! \times 2}{3!}$ [= 1680]	*M1	$\frac{7! \times k}{3!}, k = 1, 2$
	[Probability =] $\frac{\textit{their } 1680}{\textit{their } 6720}$	*M1	With denominator = *their* **(a)** or correct
	$1 - \frac{\textit{their } 1680}{\textit{their } 6720}$	DM1	$1 - m$, $0 < m < 1$, dependent on at least one M mark already gained
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
		4	

137. 9709_s21_qp_51 Q: 4

To gain a place at a science college, students first have to pass a written test and then a practical test.

Each student is allowed a maximum of two attempts at the written test. A student is only allowed a second attempt if they fail the first attempt. No student is allowed more than one attempt at the practical test. If a student fails both attempts at the written test, then they cannot attempt the practical test.

The probability that a student will pass the written test at the first attempt is 0.8. If a student fails the first attempt at the written test, the probability that they will pass at the second attempt is 0.6. The probability that a student will pass the practical test is always 0.3.

(a) Draw a tree diagram to represent this information, showing the probabilities on the branches. [3]

(b) Find the probability that a randomly chosen student will succeed in gaining a place at the college. [2]

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(c) Find the probability that a randomly chosen student passes the written test at the first attempt given that the student succeeds in gaining a place at the college. [2]

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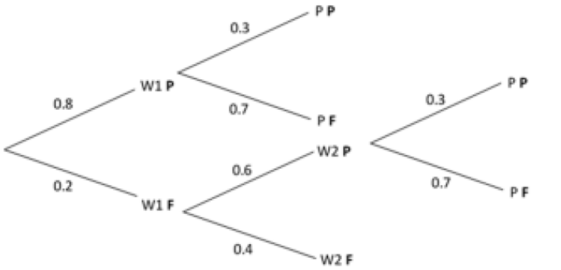
Answer:

Question	Answer	Marks	Guidance
(a)		B1	Fully correct labelled tree diagram for each pair of branches clearly identifying written and practical, pass and fail for each intersection (no additional branches)
		B1	'One written test' branch all probabilities (or %) correct
		B1	'Two written tests' branch all probabilities (or %) correct, condone additional branches after W2F with probabilities 1 for PF and 0 for PP
		3	
(b)	[P(W1P) × P(PP) + P(W1F) × P(W2P) × P(PP)] $0.8 \times 0.3 + 0.2 \times 0.6 \times 0.3$	M1	Consistent with *their* tree diagram or correct
	0.276 or $\frac{69}{250}$	A1	
		2	
(c)	$P(W1\|P) = \frac{P(W1 \cap \text{Practical})}{P(\text{getting place})} = \frac{0.8 \times 0.3}{their(b)} \left[= \frac{0.24}{0.276} \right]$	M1	Correct expression or FT *their* **(b)**
	$\frac{20}{23}$ or 0.87[0]	A1	
		2	

138. 9709_s21_qp_52 Q: 3

On each day that Alexa goes to work, the probabilities that she travels by bus, by train or by car are 0.4, 0.35 and 0.25 respectively. When she travels by bus, the probability that she arrives late is 0.55. When she travels by train, the probability that she arrives late is 0.7. When she travels by car, the probability that she arrives late is x.

On a randomly chosen day when Alexa goes to work, the probability that she does not arrive late is 0.48.

(a) Find the value of x. [3]

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(b) Find the probability that Alexa travels to work by train given that she arrives late. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	P(not late) = 0.4 × 0.45 + 0.35 × 0.3 + 0.25 × (1 – x) or P(late) = 0.4 × 0.55 + 0.35 × 0.7 + 0.25x	M1	$0.4\times p+0.35\times q+0.25\times r$, p = 0.45, 0.55, q = 0.3, 0.7 and r = (1 – x), x
	0.18 + 0.105 + 0.25 (1 – x) = 0.48 or 0.22 + 0.245 + 0.25x = 0.52	A1	Linear equation formed using sum of 3 probabilities and 0.48 or 0.52 as appropriate. Accept unsimplified.
	x = 0.22	A1	Final answer
		3	
(b)	$\left[P(train\|late)=\dfrac{P(train\cap late)}{P(late)}\right]$ $=\dfrac{0.35\times0.7}{1-0.48}$ or $\dfrac{0.35\times0.7}{0.4\times0.55+0.35\times0.7+0.25\times \textit{their}\,0.22}$	B1	0.35×0.7 or 0.245 seen as numerator of fraction
		M1	P(late) seen as a denominator with *their* probability as numerator (Accept $\dfrac{\textit{their } p}{0.52}$ or $\dfrac{\textit{their } p}{0.22+0.245+0.25\times \textit{their}\,0.22}$)
	= 0.471 or $\dfrac{49}{104}$	A1	
		3	

139. 9709_s21_qp_52 Q: 6

(a) Find the total number of different arrangements of the 8 letters in the word TOMORROW. [2]

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(b) Find the total number of different arrangements of the 8 letters in the word TOMORROW that have an R at the beginning and an R at the end, and in which the three Os are not all together. [3]

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Four letters are selected at random from the 8 letters of the word TOMORROW.

(c) Find the probability that the selection contains at least one O and at least one R. [5]

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Answer:

Question	Answer	Marks	Guidance
(a)	$\frac{8!}{2!3!}$	M1	$\frac{8!}{k! \times m!}$ $k = 1$ or 2, $m = 1$ or 3, not $k = m = 1$ no additional terms
	3360	A1	
		2	

Question	Answer	Marks	Guidance
(b)	**Method 1** Arrangements Rs at ends – Arrangements Rs at ends and Os together		
	[Os not together =] $\frac{6!}{3!} - 4!$	M1	$\frac{6!}{k!} - m$, $1 \leqslant k \leqslant 3$, m an integer, condone $2 \times \left(\frac{6!}{k!}\right) - m$.
		M1	$w - 4!$ or $w - 24$, w an integer Condone $w - 2 \times 4!$
	96	A1	
	Method 2 identified scenarios R _ _ _ R, Arrangement No Os together + 2Os and a single O		
	${}^4C_3 \times 3! + {}^4C_2 \times 2 \times 3!$	M1	${}^4C_3 \times 3! + r$ or $4 \times 3! + r$ or ${}^4P_3 \times 3! + r$, r an integer. Condone $2 \times {}^4C_3 \times 3! + r$. $2 \times 4 \times 3! + r$ or $2 \times {}^4P_3 \times 3! + r$.
		M1	$q + {}^4C_2 \times 3! \times k$ or $q + {}^4P_2 \times 3! \times k$, $k = 1,2$, q an integer
	[24 + 72 =] 96	A1	
		3	
(c)	**Method 1** Identified scenarios		
	OORR ${}^3C_2 \times {}^2C_2 \times [{}^3C_0] = 3 \times 1 = 3$ ORR_ ${}^3C_1 \times {}^2C_2 \times {}^3C_1 = 3 \times 1 \times 3 = 9$ OOR_ ${}^3C_2 \times {}^2C_1 \times {}^3C_1 = 3 \times 2 \times 3 = 18$ OR_ _ ${}^3C_1 \times {}^2C_1 \times {}^3C_2 = 3 \times 2 \times 3 = 18$ OOOR ${}^3C_3 \times {}^2C_1 \times [{}^3C_0] = 1 \times 2 = 2$	B1	Outcomes for 2 identifiable scenarios correct, accept unsimplified.
		M1	Add 4 or 5 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability = $\frac{50}{{}^8C_4}$	M1	$\frac{their\text{'}50\text{'}}{{}^8C_4}$, accept numerator unevaluated

Question	Answer	Marks	Guidance
(c) cont'd	$\frac{50}{70}$ or 0.714	A1	
	Method 2 Identified outcomes		
	ORTM $^3C_1 \times {}^2C_1 = 6$ ORTW $^3C_1 \times {}^2C_1 = 6$ ORMW $^3C_1 \times {}^2C_1 = 6$ ORRM $^3C_1 \times {}^2C_2 = 3$ ORRW $^3C_1 \times {}^2C_2 = 3$ ORRT $^3C_1 \times {}^2C_2 = 3$ OROR $^3C_2 \times {}^2C_2 = 3$ OROT $^3C_2 \times {}^2C_1 = 6$ OROM $^3C_2 \times {}^2C_1 = 6$ OROW $^3C_2 \times {}^2C_1 = 6$ OROO $^3C_3 \times {}^2C_1 = 2$	B1 M1	Outcomes for 5 identifiable scenarios correct, accept unsimplified. Add 9, 10 or 11 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability = $\frac{50}{^8C_4}$	M1	$\frac{their\text{'}50\text{'}}{^8C_4}$, accept numerator unevaluated.
	$\frac{50}{70}$ or 0.714	A1	
		5	

140. 9709_w21_qp_51 Q: 3

For her bedtime drink, Suki has either chocolate, tea or milk with probabilities 0.45, 0.35 and 0.2 respectively. When she has chocolate, the probability that she has a biscuit is 0.3. When she has tea, the probability that she has a biscuit is 0.6. When she has milk, she never has a biscuit.

Find the probability that Suki has tea given that she does not have a biscuit. [5]

Answer:

Question	Answer	Marks	Guidance
	$\left[\mathrm{P}(T\|B') = \frac{\mathrm{P}(T \cap B')}{\mathrm{P}(B')}\right]$ $\mathrm{P}(B') = 0.45 \times 0.7 + 0.35 \times 0.4 + 0.2 \times 1$	**M1**	$0.45 \times a + 0.35 \times b + 0.2[\times 1], a = 0.7, 0.3\ b = 0.4, 0.6$, seen anywhere.
	$\left[= 0.655, \frac{131}{200}\right]$	**A1**	Correct, accept unsimplified.
	$\mathrm{P}(T \cap B') = 0.35 \times 0.4$ $[= 0.14, \frac{7}{50}]$	**M1**	Seen as numerator or denominator of a fraction.
	$\mathrm{P}(T \mid B') = \frac{\textit{their}\ 0.14}{\textit{their}\ 0.655}$	**M1**	Values substituted into conditional probability formula correctly. Accept unsimplified. Denominator sum of 3 two-factor probabilities (condone omission of 1 from final factor). If clearly identified, condone from incomplete denominator.
	$0.214, \frac{28}{131}$	**A1**	If 0 marks awarded, **SC B1** 0.214 WWW.
		5	

141. 9709_w21_qp_51 Q: 5

Raman and Sanjay are members of a quiz team which has 9 members in total. Two photographs of the quiz team are to be taken.

For the first photograph, the 9 members will stand in a line.

(a) How many different arrangements of the 9 members are possible in which Raman will be at the centre of the line? [1]

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(b) How many different arrangements of the 9 members are possible in which Raman and Sanjay are not next to each other? [3]

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For the second photograph, the members will stand in two rows, with 5 in the back row and 4 in the front row.

(c) In how many different ways can the 9 members be divided into a group of 5 and a group of 4? [2]

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(d) For a random division into a group of 5 and a group of 4, find the probability that Raman and Sanjay are in the same group as each other. [4]

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Answer:

Question	Answer	Marks	Guidance
(a)	[8! =] 40 320	B1	Evaluated, exact value only.
		1	
(b)	**Method 1** [^ ^ ^ R ^ ^ S ^ ^]		
	$7! \times {}^{8}C_2 \times 2$	M1	$7! \times k$ seen, k an integer > 1.
		M1	$m \times n(n-1)$ or $m \times {}^{n}C_2$ or $m \times {}^{n}P_2$, n = 7, 8 or 9, m an integer > 1.
	282 240	A1	Exact value only. **SC B1** for final answer 282 240 WWW.
	Method 2 [Total number of arrangements – Arrangements with R & S together]		
	$9! - 8! \times 2$	M1	$9! - k$, k an integer < 362 880 .
		M1	$m - 8! \times n$, m an integer > 40 320, n = 1,2.
	282 240	A1	Exact value only. **SC B1** for final answer 282 240 WWW.
		3	
(c)	${}^{9}C_5$ [$\times {}^{4}C_4$]	M1	${}^{9}C_x$ [$\times {}^{9-x}C_{9-x}$] x = 4, 5. Condone × 1 for ${}^{9-x}C_{9-x}$. Condone use of P.
	126	A1	WWW
		2	

Question	Answer	Marks	Guidance
(d)	[Number of ways with Raman and Sanjay together on back row =] ${}^{7}C_3$ [Number of ways with Raman and Sanjay together on front row =] ${}^{7}C_2$	M1	${}^{7}C_x$ seen, x = 3 or 2.
	[Total =] 35 + 21	M1	Summing two correct scenarios.
	56	A1	Evaluated – may be seen used in probability. If **M0** scored, **SC B1** for 56 WWW.
	Probability $= \frac{\textit{their}\ 56}{\textit{their}\ (c)} = \frac{56}{126}, \frac{4}{9}, 0.444$	B1 FT	FT *their* 56 from adding 2 or more scenarios in numerator and *their* **(c)** or correct as denominator.
		4	

142. 9709_w21_qp_52 Q: 1

Each of the 180 students at a college plays exactly one of the piano, the guitar and the drums. The numbers of male and female students who play the piano, the guitar and the drums are given in the following table.

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

A student at the college is chosen at random.

(a) Find the probability that the student plays the guitar. [1]

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(b) Find the probability that the student is male given that the student plays the drums. [2]

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(c) Determine whether the events 'the student plays the guitar' and 'the student is female' are independent, justifying your answer. [2]

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Answer:

Question	Answer	Marks	Guidance
(a)	$\frac{82}{180}, \frac{41}{90}$, 0.456	B1	
		1	
(b)	$\left[\mathrm{P}(\mathrm{M}\mid\mathrm{D}) = \frac{P(M\cap D)}{P(D)}\right] = \frac{\frac{11}{180}}{\frac{20}{180}+\frac{11}{180}}$ or $\frac{0.6011}{0.1722}$	M1	*Their* identified $\frac{P(M\cap D)}{P(D)}$ *or* from data table $\frac{11}{20+11}$, accept unsimplified, condone × 180.
	$\frac{11}{31}$, 0.355	A1	Final answer.
		2	

Question	Answer	Marks	Guidance
(c)	P(F) = $\frac{100}{180}, \frac{5}{9}$, 0.5556 OE P(G) = $\frac{82}{180}, \frac{41}{90}$ 0.4556 OE	**M1**	*Their* identified P(F) × *their* identified P(G) or correct seen, can be unsimplified.
	P(F∩G) = $\frac{38}{180}, \frac{19}{90}$, 0.2111 OE $\mathrm{P(F)}\times\mathrm{P(G)} = \frac{100}{180}\times\frac{82}{180} = \frac{41}{162}$, 0.2531 OE $\left[\neq\frac{38}{180}\right]$ Not independent	**A1**	$\frac{41}{162}, \frac{38}{180}$, $\mathrm{P(F\cap G)}$ and $\mathrm{P(F)\times P(G)}$ seen with correct conclusion, WWW. Values and labels must be seen.
	Alternative method for question 1(c)		
	P(F∩G) = $\frac{38}{180}, \frac{19}{90}$, 0.2111 OE P(G) = $\frac{82}{180}, \frac{41}{90}$, 0.4556 OE	**M1**	P(F|G) (OE) unsimplified with *their* identified probs or correct
	$\mathrm{P(F\mid G)} = \frac{\frac{38}{180}}{\frac{82}{180}} = \frac{19}{41}$, 0.4634 OE $\neq \mathrm{P(F)} = \frac{100}{180}, \frac{5}{9}$, 0.5556 OE Not independent	**A1**	$\frac{19}{41}, \frac{100}{180}$, $\mathrm{P(F\cap G)}$ and $\mathrm{P(F\mid G)}$ seen with correct conclusion WWW. Values and labels must be seen.
		2	

143. 9709_w21_qp_53 Q: 5

A security code consists of 2 letters followed by a 4-digit number. The letters are chosen from $\{A, B, C, D, E\}$ and the digits are chosen from $\{1, 2, 3, 4, 5, 6, 7\}$. No letter or digit may appear more than once. An example of a code is BE3216.

(a) How many different codes can be formed? [2]

(b) Find the number of different codes that include the letter A or the digit 5 or both. [3]

A security code is formed at random.

(c) Find the probability that the code is DE followed by a number between 4500 and 5000. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	${}^5P_2 \times {}^7P_4$ **or** $5 \times 4 \times 7 \times 6 \times 5 \times 4$	**M1**	${}^5P_x \times {}^7P_y$, $1 \leqslant x \leqslant 4$, $1 \leqslant y \leqslant 6$
	16 800	**A1**	
		2	

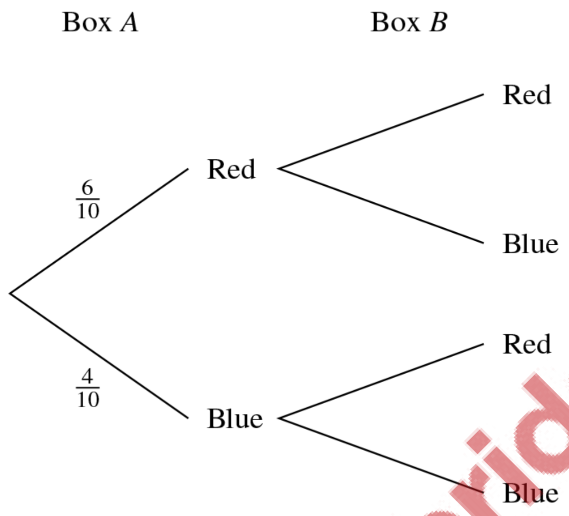
Question	Answer	Marks	Guidance
5(b)	**Method 1** [Identify scenarios]		
	With A and no 5: $8 \times {}^6P_4$ or $(1 \times 4 \times 6 \times 5 \times 4 \times 3) \times 2$ or $4C1 \times 2! \times 6P4 = 2880$ With 5 and no A: ${}^4P_2 \times 4 \times {}^6P_3$ or $(4 \times 3 \times 1 \times 6 \times 5 \times 4) \times 4$ or $4P2 \times 6C3 \times 4! = 5760$ With A and 5: $8 \times 4 \times {}^6P_3$ or $(4 \times 1 \times 1 \times 6 \times 5 \times 4) \times 8$ or $4C1 \times 2! \times 6C3 \times 4! = 3840$	**M1** **M1**	One number of ways correct, accept unsimplified. Add 2 or 3 identified correct scenarios only, accept unsimplified.
	[Total =] 12 480	**A1**	CAO
	Method 2 [total number of codes – number of codes with no A or 5]		
	No A or 5 : $(4 \times 3) \times (6 \times 5 \times 4 \times 3) = 4320$	**M1**	${}^4P_2 \times {}^6P_4$ or ${}^4C_2 \times {}^6C_4$ seen, accept unsimplified.
	Required number = *their* **(a)** – *their* 4320	**M1**	*Their* **5(a)** (or correct) – *their* (No A or 5) value.
	12 480	**A1**	
	Method 3 [subtracting **double counting**]		
	With A ${}^4P_1 \times {}^7P_4 \times 2$ or ${}^4C_1 \times 2 \times {}^7C_4 \times 4! = 6720$ With 5 ${}^5P_2 \times {}^6P_3 \times$ **4** or ${}^5C_2 \times 2 \times {}^6C_3 \times 4! = 9600$ With A and 5 = ${}^4P_1 \times {}^6P_3 \times 8$or $4C1 \times 2! \times 6C3 \times 4! \times 8 = 3840$	**M1**	One outcome correct, accept unsimplified.
	Required number = 6720 + 9600 – 3840	**M1**	Adding 'with a' to 'with 5' and subtracting 'A and 5'.
	12 480	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
(c)	**Method 1** – number of successful codes divided by total		
	$(1 \times)\ 3 \times {}^5P_2$	**M1**	$3 \times {}^5P_n$, $n = 2, 3$. Condone $3 \times {}^5C_2$, no + or –.
	Probability = $\dfrac{their\ 3 \times 5P2}{their\ 16\,800}$	**M1**	Probability = $\dfrac{their\ 60}{their\ 16\,800}$.
	$\dfrac{1}{280}$, 0.00357	**A1**	
	Method 2 – product of probabilities of each part of code		
	$\frac{1}{5} \times \frac{1}{4} \times \frac{1}{7} \times \frac{3}{6}\left(\times \frac{5}{5} \times \frac{4}{4}\right)$ or $\frac{1}{5} \times \frac{1}{4} \times \frac{3 \times 5P2}{7P4}$	**M1** **M1**	$\frac{1}{5} \times \frac{1}{4} \times k$ where $0 < k < 1$ for considering letters. $t \times \frac{1}{7} \times \frac{3}{6}$ or $t \times \frac{3 \times 5P2}{7P4}$ where $0 < t < 1$.
	$\dfrac{1}{280}$	**A1**	CAO
		3	

144. 9709_w21_qp_53 Q: 7

Box A contains 6 red balls and 4 blue balls. Box B contains x red balls and 9 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B.

(a) Complete the tree diagram below, giving the remaining four probabilities in terms of x. [3]



(b) Show that the probability that both balls chosen are blue is $\frac{4}{x+10}$. [2]

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It is given that the probability that both balls chosen are blue is $\frac{1}{6}$.

(c) Find the probability, correct to 3 significant figures, that the ball chosen from box A is red given that the ball chosen from box B is red. [5]

Answer:

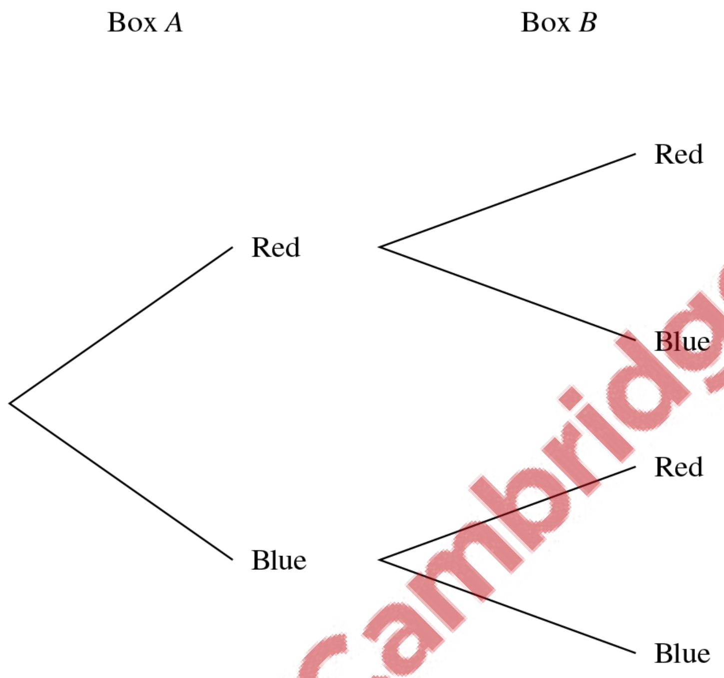
Question	Answer	Marks	Guidance
(a)	Probabilities: $\frac{x+1}{x+10}, \frac{9}{x+10}, \frac{x}{x+10}, \frac{10}{x+10}$	B1	One probability correct in correct position.
		B1	Another probability correct in correct position.
		B1	Other two probabilities correct in correct positions.
		3	
(b)	$\frac{4}{10}\times their\frac{10}{x+10}$	M1	Method consistent with *their* tree diagram.
	$\frac{4}{x+10}$	A1	AG
		2	

Question	Answer	Marks	Guidance
(c)	$\frac{4}{x+10}=\frac{1}{6}$ $x+10=24,\quad x=14$	B1	Find value of x. Can be implied by correct probabilities in calculation.
	P(ARed|BRed) = P(ARed ∩ BRed) ÷ P(BRed) $\frac{\frac{6}{10}\times their\frac{x+1}{x+10}}{\frac{6}{10}\times their\frac{x+1}{x+10}+\frac{4}{10}\times their\frac{x}{x+10}}=\frac{\frac{6}{10}\times\frac{15}{24}}{\frac{6}{10}\times\frac{15}{24}+\frac{4}{10}\times\frac{14}{24}}=\frac{\frac{3}{8}}{\frac{73}{120}}$	B1 FT	$\frac{6}{10}\times their\frac{x+1}{x+10}$ as numerator or denominator of fraction.
		M1	$\frac{6}{10}\times their\frac{x+1}{x+10}+\frac{4}{10}\times their\frac{x}{x+10}$ seen anywhere.
		A1 FT	Seen as denominator of fraction.
	$\frac{45}{73}$, 0.616[4…]	A1	If B0 M0: **SC B1** for $\frac{\frac{3}{8}}{\frac{73}{120}}$ or $\frac{0.375}{0.6083}$ **SC B1** $\frac{45}{73}$ or 0.616.
		5	

145. 9709_m20_qp_52 Q: 6

Box A contains 7 red balls and 1 blue ball. Box B contains 9 red balls and 5 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.

(a) Complete the tree diagram to show the probabilities. [3]



(b) Find the probability that the two balls chosen are not the same colour. [2]

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(c) Find the probability that the ball chosen from box A is blue given that the ball chosen from box B is blue. [4]

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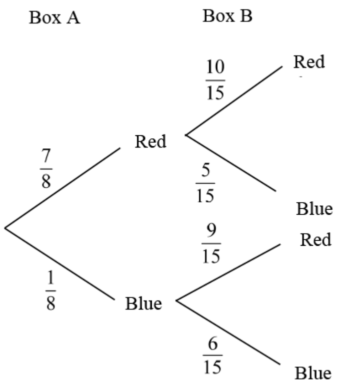
Answer:

Question	Answer	Marks	Guidance
(a)		**B1**	Both correct probs, box A
		B1	2 probs correct for box B
		B1	All correct probs for box B
		3	
(b)	$\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{9}{15}$	**M1**	Two 2 factor terms added, correct or FT *their* **6(a)**.
	$=\frac{44}{120}\left[\frac{11}{30}\text{ or }0.367\right]$	**A1**	OE
		2	

Question	Answer	Marks	Guidance
(c)	$\text{P(A blue \| B blue)}=\frac{\text{P(A blue}\cap\text{B blue)}}{\text{P(B blue)}}$ $=\frac{\frac{1}{8}\times\frac{6}{15}}{\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}}=\frac{\frac{1}{20}}{\frac{41}{120}}$	**M1**	*their* $\frac{1}{8}\times\frac{6}{15}$ seen as numerator or denom of fraction
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen as denominator
	$=\frac{6}{41}$ or 0.146	**A1**	
		4	

146. 9709_s20_qp_51 Q: 5

On Mondays, Rani cooks her evening meal. She has a pizza, a burger or a curry with probabilities 0.35, 0.44, 0.21 respectively. When she cooks a pizza, Rani has some fruit with probability 0.3. When she cooks a burger, she has some fruit with probability 0.8. When she cooks a curry, she never has any fruit.

(a) Draw a fully labelled tree diagram to represent this information. [2]

(b) Find the probability that Rani has some fruit. [2]

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(c) Find the probability that Rani does not have a burger given that she does not have any fruit. [4]

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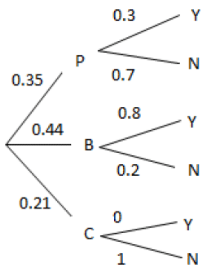
Answer:

Question	Answer	Marks
(a)		
	Fully correct labelled tree for method of transport with correct probabilities.	B1
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the prob 0	B1
		2
(b)	$0.35 \times 0.3 + 0.44 \times 0.8\ (+0)$	M1
	0.457	A1
		2

Question	Answer	Marks
(c)	P(not B|not fruit) = $\frac{P(B' \cap F')}{P(F')}$	M1
	$\frac{0.35 \times 0.7 + 0.21 \times 1}{1 - \textit{their}\,(\mathbf{b})}$	M1
	$\frac{0.455}{0.543}$ (**M1** for 1 – *their* **(b)** or summing three appropriate 2-factor probabilities, correct or consistent with *their* tree diagram as denominator)	M1
	0.838 or $\frac{455}{543}$	A1
		4

147. 9709_s20_qp_52 Q: 2

A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of students in each category are shown in the following table.

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

(a) Find the probability that a randomly chosen student is at Canton college and prefers hockey. [1]

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(b) Find the probability that a randomly chosen student is at Devar college given that he prefers soccer. [2]

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(c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justifying your answer. [2]

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Answer:

Question	Answer	Marks
(a)	$\frac{56}{500}$ or $\frac{14}{125}$ or 0.112	B1
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(b)	$P(D\mid S)=\frac{P(D\cap S)}{P(S)}=\frac{120}{280}$	M1
	$\frac{120}{280}$ or $\frac{3}{7}$	A1
		2

Question	Answer	Marks
(c)	P(hockey) = $\frac{220}{500}=0.44$ P(Amos or Benn) = $\frac{242}{500}=0.484$ P(hockey ∩ A or B) = $\frac{104}{500}=0.208$ P(H) × P(A U B) = P(H ∩ (A U B)) if independent	M1
	$\frac{220}{500}\times\frac{242}{500}=\frac{1331}{6250}$ so not independent	A1
		2

148. 9709_s20_qp_53 Q: 1

Juan goes to college each day by any one of car or bus or walking. The probability that he goes by car is 0.2, the probability that he goes by bus is 0.45 and the probability that he walks is 0.35. When Juan goes by car, the probability that he arrives early is 0.6. When he goes by bus, the probability that he arrives early is 0.1. When he walks he always arrives early.

(a) Draw a fully labelled tree diagram to represent this information. [2]

(b) Find the probability that Juan goes to college by car given that he arrives early. [4]

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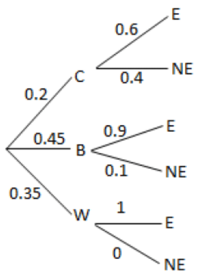
Answer:

Question	Answer	Marks
(a)	 Fully correct labelled tree for method of transport with correct probabilities.	B1
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the probability 0.	B1
		2
(b)	$P(C\|E) = \frac{P(C \cap E)}{P(E)} = \frac{0.2 \times 0.6}{0.2 \times 0.6 + 0.45 \times 0.1 + 0.35 \times 1}$	M1
	Summing three appropriate 2-factor probabilities	M1
	$\frac{0.12}{0.515}$	A1
	0.233 or $\frac{12}{515}$	A1
		4

149. 9709_s20_qp_53 Q: 7

(a) Find the number of different possible arrangements of the 9 letters in the word CELESTIAL. [1]

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(b) Find the number of different arrangements of the 9 letters in the word CELESTIAL in which the first letter is C, the fifth letter is T and the last letter is E. [2]

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(c) Find the probability that a randomly chosen arrangement of the 9 letters in the word CELESTIAL does not have the two Es together. [4]

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5 letters are selected at random from the 9 letters in the word CELESTIAL.

(d) Find the number of different selections if the 5 letters include at least one E and at most one L. [3]

Answer:

Question	Answer	Marks
(a)	$\frac{9!}{2!2!} = 90\,720$	B1
		1
(b)	$\frac{6!}{2!}$	M1
	360	A1
		2

Question	Answer	Marks
(c)	2 Es together $= \frac{8!}{2!}(= 20160)$	M1
	Es not together $= 90720 - 20160 = 70560$	M1
	Probability $= \frac{70560}{90720}$	M1
	$\frac{7}{9}$ or 0.778	A1
	Alternative method for question 7(c)	
	_^_^_^_^_^_^_^_ $\frac{7!}{2!} \times \frac{8 \times 7}{2} = 70560$	
	$7! \times k$ in numerator, k integer $\geqslant 1$, denominator $\geqslant 1$	M1
	Multiplying by $^8C_2$ OE	M1
	Probability $= \frac{70560}{90720}$	M1
	$\frac{7}{9}$ or 0.778	A1
		4

Question	Answer	Marks
(d)	Scenarios are: E L _ _ _ $\quad {}^5C_3 \quad 10$ E E L _ _ $\quad {}^5C_2 \quad 10$ E _ _ _ _ $\quad {}^5C_4 \quad 5$ E E _ _ _ $\quad {}^5C_3 \quad 10$	M1
	Summing the number of ways for 3 or 4 correct scenarios	M1
	Total = 35	A1
		3

150. 9709_w20_qp_51 Q: 1

Two ordinary fair dice, one red and the other blue, are thrown.

Event A is 'the score on the red die is divisible by 3'.

Event B is 'the sum of the two scores is at least 9'.

(a) Find $P(A \cap B)$. [2]

(b) Hence determine whether or not the events A and B are independent. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	Outcome space (table below)	M1	Complete outcome space or or listing A and B outcomes or listing $A \cap B$ outcomes
	$P(A \cap B) = \frac{5}{36}$	A1	With evidence
		2	

Blue \ Red	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Question	Answer	Marks	Guidance
(b)	$P(A) \times P(B) = \frac{1}{3} \times \frac{10}{36}$	M1	$Their \frac{1}{3} \times their \frac{10}{36}$ seen
	$\frac{5}{54} \neq \frac{5}{36}$ so not independent	A1	$\frac{5}{54}, \frac{5}{36}$, P(A) × P(B) and P(A∩*B*) seen in workings and correct conclusion stated Condone $\frac{5}{36}$ being stated in **(a)**
	Alternative method for question 1(b)		
	$P(B\|A) = P(B)$ $P(B\|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{5}{36}}{\frac{1}{3}}$	M1	OE, $\frac{their\,1(a)}{their\,P(A)}$ seen
	$\frac{5}{12} \neq \frac{5}{18}$ so not independent	A1	P(A|B), P(B), $\frac{5}{12}, \frac{5}{18}$ seen in workings and correct conclusion stated Condone $\frac{5}{18} \equiv \frac{10}{36}$ being identified in **(a)**
		2	

151. 9709_w20_qp_51 Q: 2

The probability that a student at a large music college plays in the band is 0.6. For a student who plays in the band, the probability that she also sings in the choir is 0.3. For a student who does not play in the band, the probability that she sings in the choir is x. The probability that a randomly chosen student from the college does not sing in the choir is 0.58.

(a) Find the value of x. [3]

Two students from the college are chosen at random.

(b) Find the probability that both students play in the band and both sing in the choir. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	$0{\cdot}6 \times 0{\cdot}7 + 0{\cdot}4(1 - x) = 0{\cdot}58$ $\equiv 0{\cdot}42 + 0{\cdot}4(1 - x) = 0{\cdot}58$	**M1**	Equation of form $0{\cdot}6 \times a + 0{\cdot}4 \times b = 0{\cdot}58$; $a = 0{\cdot}3, 0{\cdot}7$, $b = x, (1 - x)$
		B1	Single correct product seen, condone 0·42, in an equation of appropriate form
	$x = 0{\cdot}6$	**A1**	
	Alternative method for question 2(a)		
	$0{\cdot}6 \times 0{\cdot}3 + 0{\cdot}4x = 0{\cdot}42$ $\equiv 0{\cdot}18 + 0{\cdot}4x = 0{\cdot}42$	**M1**	Equation of form $0{\cdot}6 \times a + 0{\cdot}4 \times b = 0{\cdot}42$; $a = 0{\cdot}3, 0{\cdot}7$, $b = x, (1 - x)$
		B1	Single correct product seen, condone 0·18, in an equation of appropriate form
	$x = 0{\cdot}6$	**A1**	
		3	
(b)	$(0.6\times0.3)^2$	**M1**	$(a \times b)^2$, $a = 0{\cdot}6, 0{\cdot}4$ and $b = 0{\cdot}7, 0{\cdot}3, x, (1-x)$ or $0{\cdot}18^2$, alone.
	0.0324	**A1**	
		2	

152. 9709_w20_qp_51 Q: 7

(a) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that all 3 Es are together. [2]

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(b) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that the Ps are not next to each other. [4]

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(c) Find the probability that a randomly chosen arrangement of the 10 letters of the word SHOPKEEPER has an E at the beginning and an E at the end. [2]

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Four letters are selected from the 10 letters of the word SHOPKEEPER.

(d) Find the number of different selections if the four letters include exactly one P. [3]

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Answer:

(a)	$\frac{8!}{2!}$	**M1**	$\frac{8!}{k} \equiv \frac{7! \times 8}{k}$, where $k \in \mathbb{N}$, $\frac{a!}{2(!)}$, where $a \in \mathbb{N}$
	20160	**A1**	
		2	

Question	Answer	Marks	Guidance
(b)	Total number of ways: $\frac{10!}{2!3!}$ $(= 302\,400)$ (A)	**B1**	Accept unsimplified
	With Ps together: $\frac{9!}{3!}$ $(= 60\,480)$ (B)	**B1**	Accept unsimplified
	With Ps not together: 302 400 – 60 480	**M1**	$\frac{10!}{m} - \frac{9!}{n}$, *m, n* integers or (A) – (B) if clearly identified
	241 920	**A1**	
	Alternative method for question 7(b)		
	$\frac{8!}{3!}$	**B1**	$k \times 8!$ in numerator, k a positive integer, no ±
		B1	$m \times 3!$ in denominator, m a positive integer, no ±
	$\times \frac{9 \times 8}{2}$	**M1**	*Their* $\frac{8!}{3!}$ multiplied by ${}^9C_2$ or ${}^9P_2$ no additional terms
	241 920	**A1**	Exact value, WWW
		4	

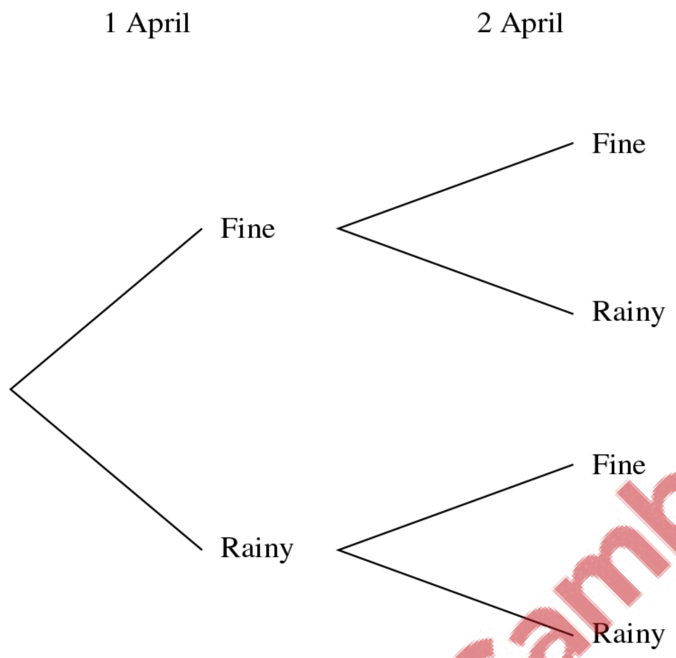
Question	Answer	Marks	Guidance
(c)	Probability = $\frac{\text{Number of ways Es at beginning and end}}{\text{Total number of ways}}$ Probability = $\frac{\frac{8!}{2!}}{\frac{10!}{2! \times 3!}} = \frac{20160}{302\,400}$	**M1**	$\frac{\left(\frac{8!}{k!}\right)}{\frac{10!}{k!l!}}$ $1 \leqslant k, l \in \mathbb{N} \leqslant 3$, FT denominator from **7(b)** or correct
	$\frac{1}{15}$, 0·0667	**A1**	
	Alternative method for question 7(c)		
	Probability = $\frac{3}{10} \times \frac{2}{9}$	**M1**	$\frac{a}{10} \times \frac{a-1}{9}$ $a = 3, 2$
	$\frac{1}{15}$, 0·0667	**A1**	
	Alternative method for question 7(c)		
	Probability = $\frac{1}{10} \times \frac{1}{9} \times 3!$	**M1**	$\frac{1}{10} \times \frac{1}{9} \times m!, m = 3, 2$
	$\frac{1}{15}$, 0.0667	**A1**	
		2	

Question	Answer	Marks	Guidance
(d)	Scenarios: P E E E ${}^5C_0 = 1$ P E E _ ${}^5C_1 = 5$ P E _ _ ${}^5C_2 = 10$ P _ _ _ ${}^5C_3 = 10$	**M1**	${}^5C_x$ seen alone, $1 \leqslant x \leqslant 4$
		M1	Summing the number of ways for 3 or 4 correct scenarios (can be unsimplified), no incorrect scenarios
	Total = 26	**A1**	
		3	

153. 9709_w20_qp_52 Q: 4

In a certain country, the weather each day is classified as fine or rainy. The probability that a fine day is followed by a fine day is 0.75 and the probability that a rainy day is followed by a fine day is 0.4. The probability that it is fine on 1 April is 0.8. The tree diagram below shows the possibilities for the weather on 1 April and 2 April.

(a) Complete the tree diagram to show the probabilities. [1]



(b) Find the probability that 2 April is fine. [2]

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Let X be the event that 1 April is fine and Y be the event that 3 April is rainy.

(c) Find the value of $P(X \cap Y)$. [3]

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(d) Find the probability that 1 April is fine given that 3 April is rainy. [3]

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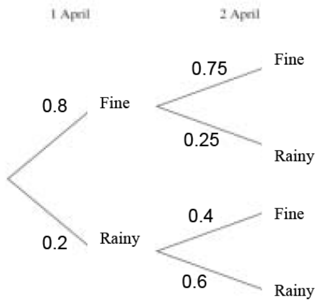
Answer:

Question	Answer	Marks	Guidance
(a)		B1	All probabilities correct, may be on branch or next to 'Fine/Rainy' Ignore additional branches.
		1	
(b)	$0{\cdot}8 \times 0{\cdot}75 + 0{\cdot}2 \times 0{\cdot}4$ $(= 0{\cdot}6 + 0{\cdot}08)$	M1	Correct or FT from *their* diagram unsimplified, all probabilities $0 < p < 1$. Partial evaluation only sufficient when correct. Accept working in **4(b)** or by the tree diagram.
	$0.68, \frac{17}{25}$	A1	From supporting working
		2	
(c)	$0{\cdot}8 \times 0{\cdot}75 \times 0{\cdot}25 + 0{\cdot}8 \times 0{\cdot}25 \times 0{\cdot}6$	M1	$a \times b \times c + a \times 1\text{-}b \times d$, $0 < c, d \leqslant 1$, a, b consistent with *their* tree diagram or correct, no additional terms
	$0{\cdot}15 + 0{\cdot}12$	A1	At least one term correct, accept unsimplified
	0.27	A1	Final answer
		3	
(d)	$P(Y) =$ *their* **(c)** $+ 0{\cdot}2 \times 0{\cdot}4 \times 0{\cdot}25 + 0{\cdot}2 \times 0{\cdot}6 \times 0{\cdot}6$ $(= 0{\cdot}362)$	B1 FT	*their* **(c)** $+ e \times f \times g + e \times (1\text{-}f) \times h$, $0 < g, h \leqslant 1$, e, f consistent with *their* tree diagram, or correct
	$P(X\|Y) = \frac{their\,(c)}{their\,P(Y)} = \frac{0.27}{0.362}$	M1	*their* **4(c)** (or correct)/*their* previously calculated and identified P(Y) or a denominator involving 3 or 4 3-factor probability terms consistent with *their* tree diagram & third factor $0 < p < 1$
	$0{\cdot}746, \frac{373}{500}$ or $\frac{135}{181}$	A1	(0.7458…)
		3	

154. 9709_w20_qp_52 Q: 6

Mr and Mrs Ahmed with their two children, and Mr and Mrs Baker with their three children, are visiting an activity centre together. They will divide into groups for some of the activities.

(a) In how many ways can the 9 people be divided into a group of 6 and a group of 3? [2]

5 of the 9 people are selected at random for a particular activity.

(b) Find the probability that this group of 5 people contains all 3 of the Baker children. [3]

All 9 people stand in a line.

(c) Find the number of different arrangements in which Mr Ahmed is not standing next to Mr Baker. [3]

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(d) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	${}^9C_6$ ($\times$ ${}^3C_3$)	M1	${}^9C_k \times n$, $k = 6, 3$, $n = 1,2$ oe Condone ${}^9C_6 + {}^3C_3$, ${}^9P_6 \times {}^3P_3$
	84	A1	Accept unevaluated.
		2	
(b)	Number with 3 Baker children = ${}^6C_2$ or 15	B1	Correct seen anywhere, not multiplied or added
	Total no of selections = ${}^9C_5$ or 126 Probability = $\frac{\text{number of selections with3 Baker children}}{\text{total number of selections}}$	M1	Seen as denominator of fraction
	$\frac{15}{126}$, 0·119	A1	OE, e.g. $\frac{5}{42}$
	Alternative method for question 6(b)		
	$\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\left(\times\frac{6}{6}\right)\left(\times\frac{5}{5}\right)\times{}^5C_3$	B1	${}^5C_3$ (OE) or 10 seen anywhere, multiplied by fractions only, not added
		M1	$\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\left(\times\frac{6}{6}\right)\left(\times\frac{5}{5}\right)\times k$, $1 \leqslant k$, k integer
	$\frac{15}{126}$, 0·119	A1	OE, e.g. $\frac{5}{42}$
		3	

Question	Answer	Marks	Guidance
(c)	[Total no of arrangements = 9!] [Arrangements with men together = 8! × 2] Not together: 9! –	M1	9! – k or 362880 – k, k an integer<362 880
	8! × 2	B1	8! × 2(!) or 80 640 seen anywhere
	282 240	A1	Exact value
	Alternative method for question 6(c)		
	7! × 8 × 7	B1	7! × k, k positive integer > 1
		M1	$m \times 8 \times 7$, $m \times {}^8P_2$, $m \times {}^8C_2$ m positive integer > 1
	282 240	A1	Exact value
		3	
(d)	7! × 2 × 7	M1	7! × k, k positive integer > 1 If 7! not seen, condone 7 × 6 × 5 × 4 × 3 × 2 × (1) × k *or* 7 × 6! × k only
		M1	$m \times 2 \times 7$, m positive integer > 1
	70 560	A1	
		3	

155. 9709_w20_qp_53 Q: 5

The 8 letters in the word RESERVED are arranged in a random order.

(a) Find the probability that the arrangement has V as the first letter and E as the last letter. [3]

(b) Find the probability that the arrangement has both Rs together given that all three Es are together. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	Total number of ways = $\frac{8!}{3!2!}$ (= 3360)	B1	Correct unsimplified expression for total number of ways
	Number of ways with V and E in correct positions = $\frac{6!}{2!\times 2!}$ (= 180)	B1	$\frac{6!}{2!\times 2!}$ alone or as numerator in an attempt to find the number of ways with V and E in correct positions. No $\times, \pm$
	Probability = $\frac{180}{3360}\left(=\frac{3}{56}\right)$ or 0.0536	B1 FT	Final answer from *their* $\frac{6!}{2!\times 2!}$ divided by *their* total number of ways
	Alternative method for question 5(a)		
	$\frac{1}{8}\times\frac{3}{7}$	M1	$\frac{a}{8}\times\frac{b}{7}$ seen, no other terms (correct denominators)
		M1	$\frac{1}{c}\times\frac{3}{d}$ seen, no other terms (correct numerators)
	$\frac{3}{56}$ or 0.0536	A1	
		3	

Question	Answer	Marks	Guidance
(b)	Rs together and Es together: 5! (120)	B1	Alone or as numerator of probability to represent the number of ways with Rs and Es together, no ×, +, –
	Es together: $\frac{6!}{2!}(=360)$	B1	Alone or as denominator of probability to represent the number of ways with Es together, no ×, + or –
	Probability = $\frac{5!}{\frac{6!}{2!}}$	M1	$\frac{their\ 5!}{their\ \frac{6!}{2!}}$ seen
	$\frac{1}{3}$	A1	OE
	Alternative method for question 5(b)		
	P(Rs together and Es together): $\frac{5!}{their\ \text{total number of ways}}\left(=\frac{1}{28}\right)$	B1	
	P(Es together): $\frac{\frac{6!}{2!}}{their\ \text{total number of ways}}\left(=\frac{3}{28}\right)$	B1	Alone or as numerator of probability to represent the P(Rs and Es together), no ×, +, –
	Probability = $\frac{\frac{1}{28}}{\frac{3}{28}}$	M1	Alone or as denominator of probability to represent the P(Es together), no ×, + or –
	$\frac{1}{3}$	A1	OE, $\frac{their\ \frac{1}{28}}{their\ \frac{3}{28}}$ seen
		4	

156. 9709_m19_qp_62 Q: 1

On each day that Tamar goes to work, he wears either a blue suit with probability 0.6 or a grey suit with probability 0.4. If he wears a blue suit then the probability that he wears red socks is 0.2. If he wears a grey suit then the probability that he wears red socks is 0.32.

(i) Find the probability that Tamar wears red socks on any particular day that he is at work. [2]

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(ii) Given that Tamar is not wearing red socks at work, find the probability that he is wearing a grey suit. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	$0.6 \times 0.2 + 0.4 \times 0.32$	**M1**	Addition of 2 two-factor terms $0.6 \times a + 0.4 \times b$
	$= 0.248, \frac{31}{125}$	**A1**	CAO
		2	
(ii)	**Method 1**		
	P(GS|Not Red socks) = $\frac{0.4 \times 0.68}{1-(i)}$	**B1**	Correct [unsimplified] numerator seen in fraction
		M1	1 – their **(i)** as denominator in fraction
	$= 0.362, \frac{17}{47}$	**A1**	
	Method 2		
	P(GS|Not Red socks) = $\frac{0.4 \times 0.68}{0.6 \times 0.8 + 0.4 \times 0.68}$	**B1**	Correct [unsimplified] numerator seen in fraction
		M1	Correct or (their **(i)**)' as denominator in fraction
	$= 0.362, \frac{17}{47}$	**A1**	
		3	

157. 9709_s19_qp_61 Q: 2

Jameel has 5 plums and 3 apricots in a box. Rosa has x plums and 6 apricots in a box. One fruit is chosen at random from Jameel's box and one fruit is chosen at random from Rosa's box. The probability that both fruits chosen are plums is $\frac{1}{4}$. Write down an equation in x and hence find x. [3]

Answer:

Question	Answer	Marks	Guidance
	Jameel: P(plum) = $\frac{5}{8}$, Rosa: P(plum) = $\frac{x}{x+6}$	**M1**	*Their* 2 probabilities for P(plum) multiplied and equated to 1/4
	$\frac{5}{8}\times\frac{x}{x+6}=\frac{1}{4}$	**A1**	Correct equation oe
	(x =) 4	**A1**	**SC** correct answer with no appropriate equations i.e. common sense **B1**
		3	

158. 9709_s19_qp_61 Q: 3

A fair six-sided die is thrown twice and the scores are noted. Event X is defined as 'The total of the two scores is 4'. Event Y is defined as 'The first score is 2 or 5'. Are events X and Y independent? Justify your answer. [4]

Answer:

Question	Answer	Marks	Guidance
	$P(X) = \frac{3}{36}\ \left(\frac{1}{12}oe\right)$	B1	
	$P(Y) = \frac{12}{36}\ \left(\frac{1}{3}oe\right)$	B1	
	$P(X \cap Y) = \frac{1}{36}$	M1	Independent method to find $P(X \cap Y)$ without multiplication, either stated or by listing or circling numbers on a probability space diagram. OR condititional prob with a single fraction numerator
	$P(X) \times P(Y) = P(X \cap Y)$, independent	A1	Numerical comparison and conclusion, www
		4	

159. 9709_s19_qp_62 Q: 1

Two ordinary fair dice are thrown and the numbers obtained are noted. Event S is 'The sum of the numbers is even'. Event T is 'The sum of the numbers is either less than 6 or a multiple of 4 or both'. Showing your working, determine whether the events S and T are independent. [4]

Answer:

Question	Answer	Marks	Guidance
	$P(S) = \frac{1}{2}$	B1	
	$P(T) = \frac{16}{36}\left(\frac{4}{9}\right)$	B1	
	$P(S \cap T) = \frac{10}{36}\left(\frac{5}{18}\right)$	M1	$P(S \cap T)$ found by multiplication scores M0 M1 awarded if *their* value is identifiable in their sample space diagram **or** Venn diagram **or** list of terms **or** probability distribution table (oe)
	$P(S)\,P(T) \neq P(S \cap T)$ so not independent	A1	8/36, 10/36 P(S) × P(T) and $P(S \cap T)$ seen in workings and correct conclusion stated, www
	Alternative method for question 1		
	$P(S) = \frac{1}{2}$	B1	
	$P(T) = \frac{16}{36}\left(\frac{4}{9}\right)$	B1	
	$P(S \cap T) = \frac{10}{36}\left(\frac{5}{18}\right)$	M1	$P(S \cap T)$ found by multiplication scores M0 M1 awarded if *their* value is identifiable in their sample space diagram **or** Venn diagram **or** list of terms **or** probability distribution table (oe)
	$P(S\|T) = \frac{10}{16}$ or $P(T\|S) = \frac{10}{18}$ $P(S\|T) \neq P(S)$ or $P(T\|S) \neq P(T)$ so not independent	A1	**Either** 18/36, 10/16,P(S) and $P(S\|T)$ seen in workings and correct conclusion stated, www **Or** 16/36, 10/18, P(T) and P(T|S) seen in workings and correct conclusion stated, www
		4	

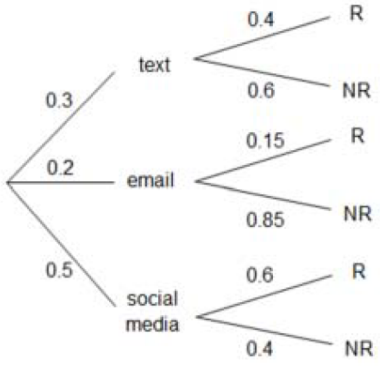
160. 9709_s19_qp_63 Q: 2

Megan sends messages to her friends in one of 3 different ways: text, email or social media. For each message, the probability that she uses text is 0.3 and the probability that she uses email is 0.2. She receives an immediate reply from a text message with probability 0.4, from an email with probability 0.15 and from social media with probability 0.6.

(i) Draw a fully labelled tree diagram to represent this information. [2]

(ii) Given that Megan does not receive an immediate reply to a message, find the probability that the message was an email. [4]

Answer:

Question	Answer	Marks	Guidance
(i)		B1	Fully correct labelled tree with correct probabilities for 'Send'
		B1	Fully correct labelled branches with correct probabilities for the 'reply'
		2	

Question	Answer	Marks	Guidance
(ii)	$P(email \mid NR) = \frac{P(\text{email} \cap \text{NR})}{P(\text{NR})} = \frac{0.2 \times 0.85}{0.3 \times 0.6 + 0.2 \times 0.85 + 0.5 \times 0.4}$	M1	P(email) × P(NR) seen as numerator of a fraction, consistent with *their* tree diagram
	$= \frac{0.17}{0.18 + 0.17 + 0.2} = \frac{0.17}{0.55}$	M1	Summing three appropriate 2-factor probabilities, consistent with *their* tree diagram, seen anywhere 0.55 oe (can be unsimplified) seen as denom of a fraction
	$= 0.309, \frac{17}{55}$	A1	
		A1	Correct answer
		4	

161. 9709_w19_qp_61 Q: 1

When Shona goes to college she either catches the bus with probability 0.8 or she cycles with probability 0.2. If she catches the bus, the probability that she is late is 0.4. If she cycles, the probability that she is late is x. The probability that Shona is not late for college on a randomly chosen day is 0.63. Find the value of x. [3]

Answer:

Question	Answer	Marks	Guidance
	$0.8 \times 0.6 + 0.2(1-x) = 0.63$	**M1**	Equation of form $0.8 \times A + 0.2 \times B = C$, A,B involving $1-x$ and 0.6 or 0.4 and C = 0.63 or 0.37
	$0.2x = 0.05$	**M1**	Correct unsimplified equation
	$x = 0.25$	**A1**	
	Alternative method for question 1		
	$0.8 \times 0.4 + 0.2x = 1 - 0.63$	**M1**	Equation of form $0.8 \times A + 0.2 \times B = C$, A,B involving x and 0.6 or 0.4 and C = 0.63 or 0.37
	$0.2x = 0.05$	**M1**	Correct unsimplified equation
	$x = 0.25$	**A1**	
		3	

162. 9709_w19_qp_62 Q: 2

Benju cycles to work each morning and he has two possible routes. He chooses the hilly route with probability 0.4 and the busy route with probability 0.6. If he chooses the hilly route, the probability that he will be late for work is x and if he chooses the busy route the probability that he will be late for work is $2x$. The probability that Benju is late for work on any day is 0.36.

(i) Show that $x = 0.225$. [2]

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(ii) Given that Benju is not late for work, find the probability that he chooses the hilly route. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	$0.4x + 0.6 \times 2x = 0.36$ or $0.4(1-x) + 0.6(1-2x) = 0.64$	**M1**	$0.4a + (1-0.4)b = 0.36$ or 0.64, a,b terms involving x
	$1.6x = 0.36$ $x = 0.225$	**A1**	Fully justified by algebra AG
		2	

Question	Answer	Marks	Guidance
(ii)	$\mathrm{P}(\mathrm{H} \mid \mathrm{L}') =$ $\frac{0.4(1-x)}{1-0.36} = \frac{0.4\times(1-0.225)}{0.64} = \frac{0.4\times 0.775}{0.4\times 0.775 + 0.6\times 0.55}$	**M1**	Correct numerical numerator of a fraction. Allow unsimplified.
		M1	Denominator 0.36 or 0.64. Allow unsimplified.
	$\frac{31}{64}$ or 0.484	**A1**	
		3	

163. 9709_w19_qp_62 Q: 7

(i) Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that all three Os are together and both Ts are together. [1]

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(ii) Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that the Ts are not together. [4]

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(iii) Find the probability that a randomly chosen arrangement of the 9 letters of the word TOADSTOOL has a T at the beginning and a T at the end. [2]

(iv) Five letters are selected from the 9 letters of the word TOADSTOOL. Find the number of different selections if the five letters include at least 2 Os and at least 1 T. [4]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)	$6! = 720$	B1	Evaluated
		1	
(ii)	Total no of arrangements: $\frac{9!}{2!3!} = 30240$	B1	Accept unevaluated
	No with Ts together $= \frac{8!}{3!} = 6720$	B1	Accept unevaluated
	With Ts not together: 30 240 – 6720	M1	correct or $\frac{9!}{m} - \frac{8!}{n}, m, n$ integers > 1 or *their* identified total – *their* identified Ts together
	23 520	A1	CAO
	Alternative method for question 7(ii)		
	$\frac{7!}{3!} \times \frac{8 \times 7}{2}$	B1	$7! \times (k > 0)$ in numerator, cannot be implied by $^7P_2$, etc.
		B1	$3! \times (k > 0)$ in denominator
		M1	$\frac{\textit{their } 7!}{\textit{their } 3!} \times {}^8C_2$ or $^8P_2$
	23 520	A1	CAO
		4	

Question	Answer	Marks	Guidance
(iii)	Number of arrangements $= \frac{7!}{3!}$ Probability $= \frac{\textit{their } \frac{7!}{3!}}{\textit{their } \frac{9!}{3!2!}} = \frac{840}{30240}$	M1	$\frac{\textit{their} \text{ identified number of arrangements with T at ends}}{\textit{their} \text{ identified total number of arrangements}}$ or $\frac{\frac{7!}{m}}{\frac{9!}{n}}$ m, n integers > 1
	$\frac{1}{36}$ or 0.0278	A1	Final answer
		2	
(iv)	OOT_ _ $\quad {}^4C_2 = 6$ OOTT_ $\quad {}^4C_1 = 4$ OOOT_ $\quad {}^4C_1 = 4$ OOOTT $\quad = 1$	M1	^4C_x seen alone or ^4C_x x $k \geq 1$, k an integer, $0 < x < 4$
		A1	$^4C_2$ x k, $k = 1$ oe or $^4C_1$ x m, $m = 1$ oe alone
		M1	Add 3 or 4 identified correct scenarios only, accept unsimplified
	(Total) = 15	A1	CAO, WWW Only dependent on 2nd M mark
		4	

164. 9709_w19_qp_63 Q: 1

There are 300 students at a music college. All students play exactly one of the guitar, the piano or the flute. The numbers of male and female students that play each of the instruments are given in the following table.

	Guitar	Piano	Flute
Female students	62	35	43
Male students	78	40	42

(i) Find the probability that a randomly chosen student at the college is a male who does not play the piano. [1]

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(ii) Determine whether the events 'a randomly chosen student is male' and 'a randomly chosen student does not play the piano' are independent, justifying your answer. [2]

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Answer:

Question	Answer	Marks	Guidance
(i)	$\frac{120}{300} = 0.4$	B1	OE
		1	
(ii)	P(male) × P(not piano) = $\frac{160}{300} \times \frac{225}{300} \left(\frac{8}{15} \times \frac{3}{4}\right) = \frac{2}{5}$	M1	P(M) × P(P') seen Can be unsimplified but the events must be named in a product
	As P(male ∩ not piano) also = $\frac{120}{300} = \frac{2}{5}$ The events are **Independent**	A1	Numerical comparison and correct conclusion
	Alternative method for question 1(ii)		
	P(male ∩ not piano) = $\frac{120}{300}$; P(not piano) = $\frac{225}{300}$	M1	P(M|P') or P(P'|M) unsimplified seen with *their* probs with correctly named events
	P(M | not piano) = $\frac{\frac{120}{300}}{\frac{225}{300}} = \frac{120}{225} = \frac{8}{15}$ = P(male) **or** P(not piano | M) = $\frac{\frac{120}{300}}{\frac{160}{300}} = \frac{120}{160} = \frac{3}{4}$ = P(not piano) Therefore the events are **Independent**	A1	Numerical comparison with P(M) or P(P') and correct conclusion
		2	

165. 9709_m18_qp_62 Q: 3

Last Saturday, Sarah recorded the colour and type of 160 cars in a car park. All the cars that were not red or silver in colour were grouped together as 'other'. Her results are shown in the following table.

		Type of car		
		Saloon	Hatchback	Estate
Colour of car	Red	20	40	12
	Silver	14	26	10
	Other	6	24	8

(i) Find the probability that a randomly chosen car in the car park is a silver estate car. [1]

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(ii) Find the probability that a randomly chosen car in the car park is a hatchback car. [1]

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(iii) Find the probability that a randomly chosen car in the car park is red, given that it is a hatchback car. [2]

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(iv) One of the cars in the car park is chosen at random. Determine whether the events 'the car is a hatchback car' and 'the car is red' are independent, justifying your answer. [2]

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Answer:

Question	Answer	Marks	Guidance
(i)	(10/160 =) 1/16, 0.0625	**B1**	OE
		1	
(ii)	(90/160) = 9/16, 0.5625	**B1**	OE
		1	
(iii)	P(red/hatchback) = P(red hatchback) / P(hatchback) = 40/160 / 90/160	**M1**	Appropriate probabilities in a fraction
	= 4/9	**A1**	OE *Altn method: Direct from table* *M1 for 40/*a *or* b*/90,* a ≠ *160* *A1 for 40/90 oe*
		2	

Question	Answer	Marks	Guidance
(iv)	*EITHER:* P(red) × P(hatchback) = $\frac{72}{160}\times\frac{90}{160}\neq\frac{40}{160}$	**(M1**	Use correct approach with appropriate probabilities substituted
	Not independent	**A1)**	Numerical comparison and conclusion stated
	OR: P(red/hatchback) = 40/90 and $\frac{40}{90}\neq\frac{72}{160}$	**(M1**	Use correct approach with appropriate probabilities substituted
	Not independent	**A1)**	Numerical comparison and conclusion stated
		2	

166. 9709_s18_qp_62 Q: 2

In a group of students, $\frac{3}{4}$ are male. The proportion of male students who like their curry hot is $\frac{3}{5}$ and the proportion of female students who like their curry hot is $\frac{4}{5}$. One student is chosen at random.

(i) Find the probability that the student chosen is either female, or likes their curry hot, or is both female and likes their curry hot. [4]

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(ii) Showing your working, determine whether the events 'the student chosen is male' and 'the student chosen likes their curry hot' are independent. [2]

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Answer:

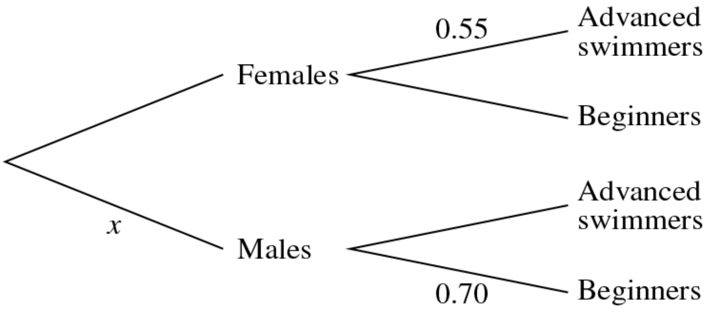
Question	Answer	Marks	Guidance
(i)	**Method 1** $P(M \cap H) = \frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$ (0.45)	**B1**	Seen, accept unsimplified
	$P(F \text{ or } M \cap H) = \frac{1}{4} + \frac{9}{20} = \frac{14}{20}$	**M1**	Numerical attempt at $P(F) + P(M \cap H)$
		A1	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) OE	**A1**	Correct final answer
	Method 2 $P(M \cap H') = \frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$ (0.3)	**B1**	Seen, accept unsimplified
	$P(F \text{ or } M \cap H) = 1 - P(M \cap H')$	**M1**	Numerical attempt at $1 - P(M \cap H')$
	$= 1 - \frac{3}{4} \times \frac{2}{5}$	**A1**	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) OE	**A1**	Correct final answer

Question	Answer	Marks	Guidance
(i)	**Method 3** $P(F \cap H' \text{ or } H) = \frac{1}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{4}{5} + \frac{3}{4} \times \frac{3}{5}$	**B1**	$\frac{3}{4} \times \frac{3}{5}$ $(\frac{9}{20})$ **or** $\frac{1}{4} \times \frac{4}{5}$ $(\frac{4}{20})$ **or** $\frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{4}{5}$ $(\frac{13}{20})$ seen
	$= \frac{1}{20} + \frac{4}{20} + \frac{9}{20}$	**M1**	Numerical attempt at $P(F \cap H') + P(F \cap H) + P(M \cap H)$
		A1	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) oe	**A1**	Correct final answer
	Method 4 – Venn diagram style approach $P(F \cup H) = P(F) + P(H) - P(F \cap H)$	**B1**	$\frac{3}{4} \times \frac{3}{5}$ $(\frac{9}{20})$ **or** $\frac{1}{4} \times \frac{4}{5}$ $(\frac{4}{20})$ **or** $\frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{4}{5}$ $(\frac{13}{20})$ seen
	$= \frac{1}{4} + \frac{1}{4} \times \frac{4}{5} + \frac{3}{4} \times \frac{3}{5} - \frac{1}{4} \times \frac{4}{5}$	**M1**	Numerical attempt at $P(F) + P(H) - P(F \cap H)$
	$= \frac{1}{4} + \frac{4}{20} + \frac{9}{20} - \frac{4}{20}$	**A1**	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) oe	**A1**	Correct final answer
		4	

Question	Answer	Marks	Guidance
(ii)	**Method 1** $(P(M) \times P(H) =)\ \frac{3}{4} \times \textit{their}\ \frac{13}{20} = \frac{39}{80}$ $(P(M \cap H) =)\ \frac{3}{4} \times \frac{3}{5} = 0.45$	**M1**	Unsimplified, or better, legitimate numerical attempt at $P(M) \times P(H)$ **and** $P(M \cap H)$ Descriptors $P(M \cap H)$ **and** $P(M) \times P(H)$ seen, correct numerical evaluation and comparison, conclusion stated
	$\frac{39}{80}$ $(0.4875) \neq 0.45$, not independent	**A1**	
	Method 2 $P(M \mid H) = \frac{P(M \cap H)}{P(H)} = \frac{\frac{9}{20}}{\text{their } \frac{13}{20}} = \frac{9}{13}$ $P(M) = \frac{3}{4}$	**M1**	Unsimplified, or better, numerical attempt at $P(H)$ and $P(M \cap H)$, $P(M)$
	$\frac{9}{13} \neq \frac{3}{4}$, not independent	**A1**	Descriptors $P(M \cap H)$, $P(H)$ **and** $P(M)$ **OR** $P(M \mid H)$ **and** $P(M)$ seen, numerical evaluation **and** comparison, conclusion stated Any appropriate relationship can be used, the M is awarded for an unsimplified, or better, numerical attempt at the terms required, the A mark requires the correct descriptors, numerical evaluation and comparison and the conclusion
		2	

167. 9709_s18_qp_63 Q: 3

The members of a swimming club are classified either as 'Advanced swimmers' or 'Beginners'. The proportion of members who are male is x, and the proportion of males who are Beginners is 0.7. The proportion of females who are Advanced swimmers is 0.55. This information is shown in the tree diagram.



For a randomly chosen member, the probability of being an Advanced swimmer is the same as the probability of being a Beginner.

(i) Find x. [3]

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(ii) Given that a randomly chosen member is an Advanced swimmer, find the probability that the member is male. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	$(1-x)$ and 0.45 (or 0.3)	**B1**	Seen, either on tree diagram or elsewhere
	Beginners: $0.7 \times x +$ '0.45' $\times$ '$(1-x)$' = 0.5 Or Advanced: '0.3' $\times x + 0.55 \times$ '$(1-x)$' = 0.5 Or $0.7 \times x +$ '0.45'$\times$ '$(1-x)$' = '0.3' $\times x + 0.55\times$ '$(1-x)$'	**M1**	One of the three correct probability equations
	$x = 0.2$ oe	**A1**	Correct answer
	Total:	**3**	
(ii)	$P(M \mid A) = \frac{P(M \cap A)}{P(A)} = \frac{0.2 \times 0.3}{0.5}$	**M1**	'i' $\times$ 0.3 as num or denom of a fraction
		M1	0.5 (or (1 – 'i') $\times$ 0.55 + 'i' $\times$ 0.3 unsimplified) seen as denom of a fraction
	$= 0.12 \left(\frac{3}{25}\right)$	**A1**	Correct answer
	Total:	**3**	

168. 9709_w18_qp_61 Q: 7

In a group of students, the numbers of boys and girls studying Art, Music and Drama are given in the following table. Each of these 160 students is studying exactly one of these subjects.

	Art	Music	Drama
Boys	24	40	32
Girls	15	12	37

(i) Find the probability that a randomly chosen student is studying Music. [1]

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(ii) Determine whether the events 'a randomly chosen student is a boy' and 'a randomly chosen student is studying Music' are independent, justifying your answer. [2]

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(iii) Find the probability that a randomly chosen student is not studying Drama, given that the student is a girl. [2]

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(iv) Three students are chosen at random. Find the probability that exactly 1 is studying Music and exactly 2 are boys. [5]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)	52/160 = 13/40, 0.325	**B1**	oe
		1	
(ii)	P(boy) = 96/160: P(Music) = 52/160 P(boy and Music) = 40/160	**M1**	Use of P(B) × P(M) = P(B∩M), appropriate probabilities used
	96/160 × 52/160 ≠ 40/160: Not independent	**A1**	Numerical comparison and conclusion stated
		2	

Question	Answer	Marks	Guidance
(iv)	**Method 2**		
	$\dfrac{\binom{40}{1}\times\binom{56}{1}\times\binom{52}{1}+\binom{12}{1}\times\binom{56}{2}}{\binom{160}{3}}$	**M1**	One scenario identified with 2 or 3 combination multiplied
		A1	One scenario correct
		B1	Denominator correct
	$\dfrac{116480+18480}{669920}$	**M1**	Both scenarios attempted, and added, seen as a numerator of a fraction
	$\dfrac{1687}{8374}$	**A1**	Correct answer, oe
		5	

169. 9709_w18_qp_62 Q: 1

(i) How many different arrangements are there of the 11 letters in the word MISSISSIPPI? [2]

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ii) Two letters are chosen at random from the 11 letters in the word MISSISSIPPI. Find the probability that these two letters are the same. [3]

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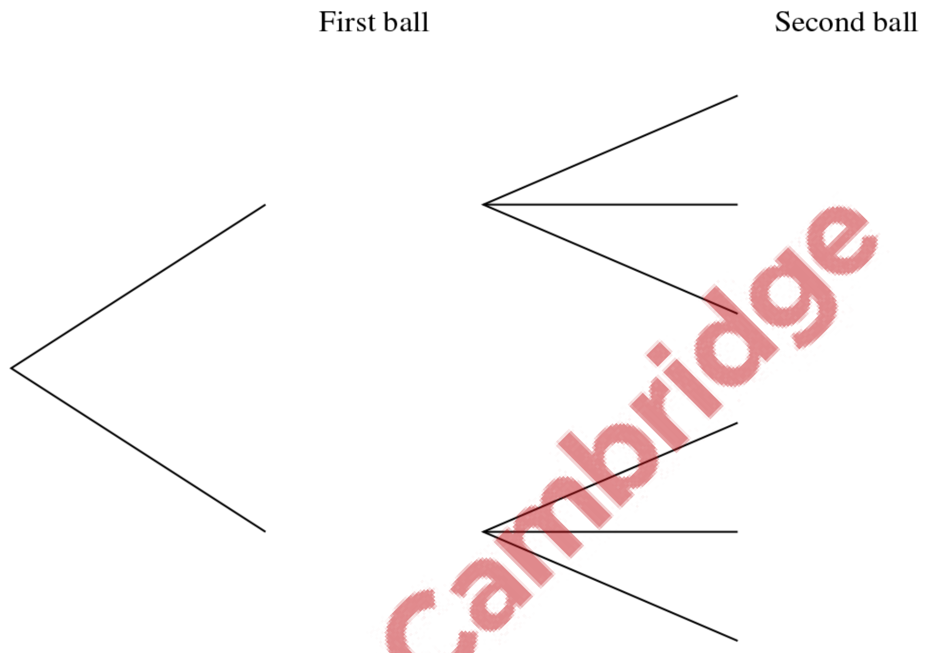
Answer:

Question	Answer	Marks	Guidance
(i)	$\frac{11!}{4!4!2!}$	M1	$\frac{11!}{4!\times k}$ *or* $\frac{11!}{2!\times k}$, k a positive integer
	= 34650	A1	Correct final answer
		2	
(ii)	**Method 1**		
	P(SS) = $\frac{4}{11}\times\frac{3}{10}=\frac{12}{110}$ (= 0.10911)	B1	One of P(SS), P(PP) or P(II) correct, allow unsimplified
	P(PP) = $\frac{2}{11}\times\frac{1}{10}=\frac{2}{110}$ (= 0.01818) P(II) = $\frac{4}{11}\times\frac{3}{10}=\frac{12}{110}$ (= 0.10911) $\frac{4}{11}\times\frac{3}{10}$	M1	Sum of probabilities from 3 appropriate identifiable scenarios (either by labelling or of form $\frac{4}{11}\times\frac{a}{b}+\frac{2}{11}\times\frac{c}{b}+\frac{4}{11}\times\frac{a}{b}$ where a = 4 *or* 3, b = 11 *or* 10, c = 2 *or* 1)
	Total = $\frac{26}{110}=\frac{13}{55}$ oe (0.236)	A1	Correct final answer
	Method 2		
	Total number of selections = $^{11}C_2$ = 55 Selections with 2 Ps = 1	B1	Seen as the denominator of fraction (no extra terms) allow unsimplified
	Selections with 2 Ss = $^4C_2$ = 6 Selections with 2 Is = $^4C_2$ = 6,	M1	Sum of 3 appropriate identifiable scenarios (either by labelling or values, condone use of permutations. May be implied by 2,12,12)
	Total selections with 2 letters the same = 13 Probability of 2 letters the same = $\frac{13}{55}$ oe (0.236)	A1	Correct final answer, without use of permutations
		3	

170. 9709_w18_qp_63 Q: 3

A box contains 3 red balls and 5 blue balls. One ball is taken at random from the box and not replaced. A yellow ball is then put into the box. A second ball is now taken at random from the box.

(i) Complete the tree diagram to show all the outcomes and the probability for each branch. [2]



(ii) Find the probability that the two balls taken are the same colour. [2]

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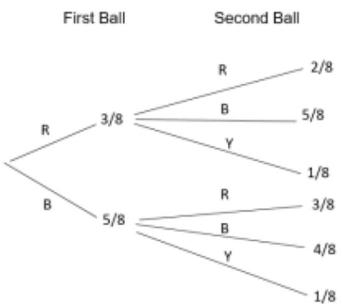
(iii) Find the probability that the first ball taken is red, given that the second ball taken is blue. [3]

Answer:

Question	Answer	Marks	Guidance
(i)		**B1**	Fully correct labelled tree and correct probabilities for 'First Ball'
		B1	Correct probabilities (with corresponding labels) for 'Second Ball'
		2	
(ii)	P(RR) + P(BB) = 3/8 × 2/8 + 5/8 × 4/8 = 3/32 + 5/16	**M1**	Correct unsimplified expression from their tree diagram, $\Sigma p = 1$ on each branch
	= 13/32 (0.406)	**A1**	Correct answer
		2	

Question	Answer	Marks	Guidance
(iii)	P(RB) = $3/8 \times 5/8$ = 15/64	**M1**	$P(\text{1st ball red}) \times P(\text{2nd ball blue})$ from their tree diagram seen unsimplified as numerator or denominator of a fraction Allow $\Sigma p \neq 1$ on each branch
	P(B) = 3/8 × 5/8 + 5/8 × 4/8 = 35/64	**M1**	Correct unsimplified expression for P(B) from their tree diagram seen as denominator of a fraction. Allow $\Sigma p \neq 1$ on each branch
	P(R|B) = P(RB) / P(B) = (15/64) ÷ (35/64) = 3/7 (0.429)	**A1**	Correct answer
		3	

171. 9709_w18_qp_63 Q: 4

Out of a class of 8 boys and 4 girls, a group of 7 people is chosen at random.

(i) Find the probability that the group of 7 includes one particular boy. [3]

(ii) Find the probability that the group of 7 includes at least 2 girls. [4]

Answer:

Question	Answer	Marks	Guidance
(i)	Total number of selections = ${}^{12}C_7 = 792$	B1	Seen as denominator of fraction
	Selections with boy included = ${}^{11}C_6$ or ${}^{12}C_7 - {}^{11}C_7 = 462$	M1	Correct unsimplified expression for selections with boy included seen as numerator of fraction
	Probability = 462/792 = 7/12 (0.583)	A1	Correct answer
	OR		
	prob of boy not included = 11/12 × 10/11 × …. × 5/6= 5/12	B1	Correct unsimplified prob
	1 – 5/12	M1	Subtracting prob from 1
	= 7/12	A1	Correct answer
		3	

Question	Answer	Marks	Guidance
(ii)	**Method 1**		
	Scenarios are: 2G + 5B: ${}^4C_2 \times {}^8C_5 = 336$	B1	One unsimplified product correct
	3G + 4B: ${}^4C_3 \times {}^8C_4 = 280$ 4G + 3B: ${}^4C_4 \times {}^8C_3 = 56$	M1	No of selections (products of ${}^nC_r$ and ${}^nP_r$) added for 2, 3 and 4 girls with no of girls and no of boys summing to 7
	Total = 672	A1	Correct total
	Probability = 672/792 (28/33) (0.848)	A1ft	Correct answer – 'total'/('total no of selections' from i)
	Method 2		
	0G + 7B ${}^4C_0 \times {}^8C_7 = 8$	B1	One unsimplified no of selections correct
	1G + 6B ${}^4C_1 \times {}^8C_6 = 112$ Total = 8 + 112 = 120	M1	No of selections (products of ${}^nC_r$ and ${}^nP_r$) added for 0 and 1 girls with no of girls and no of boys summing to 7
	$({}^{12}C_7 - 120)/792$ or 1 – 120/792	A1	792 – 120 = 672 or 1 – 120/792
	Probability = 672/792 (28/33) (0.848)	A1ft	'672' over '792' from i
	Method 3 (probability)		
	1 – P(0) – P(1) = 1 – (8/12 × 7/11 × …… × 2/6) – (8/12 × … × 3/7 × 4/6 × 7)	B1	One correct unsimplified prob for 0 or 1
	= 1 – 1/99 –14/99	M1	Subtracting 'P(0)' and 'P(1)' (using products of 7 fractions with denominators from 12 to 6) from 1
		A1	Both probs correct unsimplified
	= 84/99 = 28/33	A1ft	1 – 'P(0)' – 'P(1)'

Question	Answer	Marks	Guidance
(ii)	**Method 4 (probability)**		
	P(2) + P(3) + P(4) =	B1	One correct unsimplified prob for 2, 3 or 4
	42/99 + 35/99 + 7/99	M1	Adding 'P(2)', 'P(3)' and P(4)' (using products of 7 fractions with denominators from 12 to 6)
		A1	Three probs correct unsimplified
	= 84/99 = 28/33	A1ft	'P(2)'+ 'P(3)' + 'P(4)'
		4	

172. 9709_s17_qp_61 Q: 2

Ashfaq throws two fair dice and notes the numbers obtained. R is the event 'The product of the two numbers is 12'. T is the event 'One of the numbers is odd and one of the numbers is even'. By finding appropriate probabilities, determine whether events R and T are independent. [5]

Answer:

Question	Answer	Marks	Guidance
	$P(R) = 4/36 = 1/9$	M1	Attempt at P(*R*) by probability space diag or listing more than half the options, must see a prob, just a list is not enough
	$P(T) = P(O, E) + P(E, O) = 1/4 + 1/4 = 1/2$ OR $P(R\mid T) = 1/9$	M1	Attempt at P(*T*) or P(*R*|*T*) involving more than half the options
	$P(R \cap T) = P(3, 4) + P(4, 3) = 2/36 = 1/18$ OR $P(R\mid T) = 1/9$	B1	Value stated, not from P(R) × P(T) e.g. from probability space diagram
	As $P(R) \times P(T) = P(R \cap T)$ OR as $P(R\mid T) = P(R)$	M1	Comparing product values with $P(R \cap T)$, or comparing P (*R*|*T*) with P(*R*)
	The events are independent.	A1	Correct conclusion must have all probs correct
	Total:	**5**	

173. 9709_s17_qp_61 Q: 3

Redbury United soccer team play a match every week. Each match can be won, drawn or lost. At the beginning of the soccer season the probability that Redbury United win their first match is $\frac{3}{5}$, with equal probabilities of losing or drawing. If they win the first match, the probability that they win the second match is $\frac{7}{10}$ and the probability that they lose the second match is $\frac{1}{10}$. If they draw the first match they are equally likely to win, draw or lose the second match. If they lose the first match, the probability that they win the second match is $\frac{3}{10}$ and the probability that they draw the second match is $\frac{1}{20}$.

(i) Draw a fully labelled tree diagram to represent the first two matches played by Redbury United in the soccer season. [2]

(ii) Given that Redbury United win the second match, find the probability that they lose the first match. [4]

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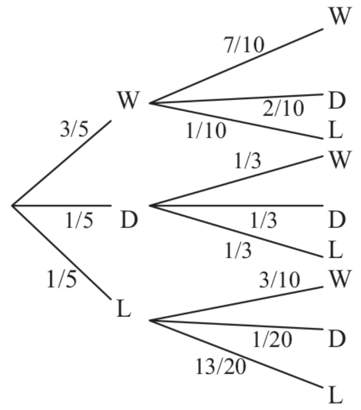
Answer:

Question	Answer	Marks	Guidance
(i)		M1	Correct shape i.e. 3 branches then 3 by 3 branches, labelled and clear annotation Condone omission of lines for first match result providing the probabilities are there.
		A1	All correct probs with fully correct shape and probs either fractions or decimals not 1.5/5 etc.
	Total:	**2**	

Question	Answer	Marks	Guidance
(ii)	$\text{P}(L_1 \text{ given } W_2) = \frac{\text{P}(L_1 \cap W_2)}{\text{P}(W_2)}$	M1	Attempt at P(L1∩W2) as a two-factor prod only as num or denom of a fraction
	$= \frac{1/5 \times 3/10}{3/5 \times 7/10 + 1/5 \times 1/3 + 1/5 \times 3/10}$	M1	Attempt at P(W2) as sum of appropriate 3 two-factor probs OE seen anywhere
		A1	Unsimplified correct P(W2) num or denom of a fraction
	$= \frac{3/50}{41/75} = 9/82 (0.110)$	A1	
	Total:	**4**	

174. 9709_s17_qp_63 Q: 1

A biased die has faces numbered 1 to 6. The probabilities of the die landing on 1, 3 or 5 are each equal to 0.1. The probabilities of the die landing on 2 or 4 are each equal to 0.2. The die is thrown twice. Find the probability that the sum of the numbers it lands on is 9. [4]

Answer:

Question	Answer	Marks	Guidance
	P(6) = 0.3	B1	SOI
	P(sum is 9) = P(3, 6) + P(4, 5) + P(5, 4) + P(6, 3)	M1	Identifying the four ways of summing to 9 (3,6), (6,3) (4,5) and (5,4)
	= (0.03 + 0.02) × 2	M1	Mult 2 probs together to find one correct prob of (3,6), (6,3) (4,5) or (5,4) unsimplified
	= 0.1	A1	OE
	Total:	**4**	

175. 9709_s17_qp_63 Q: 3

A shop sells two makes of coffee, Café Premium and Café Standard. Both coffees come in two sizes, large jars and small jars. Of the jars on sale, 65% are Café Premium and 35% are Café Standard. Of the Café Premium, 40% of the jars are large and of the Café Standard, 25% of the jars are large. A jar is chosen at random.

(i) Find the probability that the jar is small. [2]

(ii) Find the probability that the jar is Café Standard given that it is large. [3]

Answer:

(i)	P(S) = 0.65 × 0.6 + 0.35 × 0.75	**M1**	Summing two 2-factor probs or 1 – (sum of two 2-factor probs)
	= 0.653 (261/400)	**A1**	
	Total:	**2**	

Question	Answer	Marks	Guidance
(ii)	$\mathrm{P}(Std \mid L) = \frac{P(Std \cap L)}{P(L)} = \frac{0.35 \times 0.25}{1 - 0.6525}$ = 0.0875/0.3475	**M1** **M1**	'P(Std)' × 'P(L/Std)'as num of a fraction. Could be from tree diagram in 3(i). Denominator (1 - their (i)) or their (i) **or** 0.65 × 0.4(or 0.6) + 0.35 × 0.25(or 0.75) = 0.26+0.0875 **or** P(L) from their tree diagram
	= 0.252 (35/139)	**A1**	
	Total:	**3**	

176. 9709_w17_qp_61 Q: 5

Over a period of time Julian finds that on long-distance flights he flies economy class on 82% of flights. On the rest of the flights he flies first class. When he flies economy class, the probability that he gets a good night's sleep is x. When he flies first class, the probability that he gets a good night's sleep is 0.9.

(i) Draw a fully labelled tree diagram to illustrate this situation. [2]

The probability that Julian gets a good night's sleep on a randomly chosen flight is 0.285.

(ii) Find the value of x. [2]

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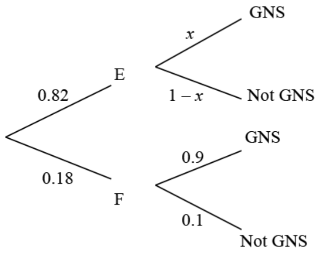
(iii) Given that on a particular flight Julian does not get a good night's sleep, find the probability that he is flying economy class. [3]

Answer:

Question	Answer	Marks	Guidance
(i)		B1	Must see at least 4 probs correct including one with an x in, correct shape
		B1	Shape, clear labels/annotation and all probs correct
		2	
(ii)	$0.82x + 0.18 \times 0.9 = 0.285$	M1	Eqn with x in , two 2-factors on one side
	$x = 0.15$	A1	
		2	
(iii)	$P(E \mid notGNS) = \frac{P(E \cap \text{notGNS})}{P(\text{notGNS})}$	M1	Attempt at P(E∩not GNS) seen as num or denom of fraction
		M1	Attempt at P(not GNS) seen anywhere
	$= \frac{0.82 \times 0.85}{1 - 0.285} = 0.975$	A1	Correct answer
		3	

177. 9709_w17_qp_63 Q: 3

At the end of a revision course in mathematics, students have to pass a test to gain a certificate. The probability of any student passing the test at the first attempt is 0.85. Those students who fail are allowed to retake the test once, and the probability of any student passing the retake test is 0.65.

(i) Draw a fully labelled tree diagram to show all the outcomes. [2]

(ii) Given that a student gains the certificate, find the probability that this student fails the test on the first attempt. [4]

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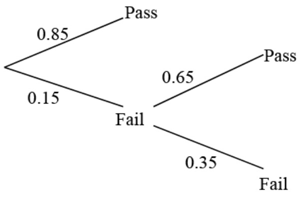
Answer:

Question	Answer	Marks	Guidance
(i)		**M1**	Correct shape
		A1	All correct labels and probabilities
		2	

Question	Answer	Marks	Guidance
(ii)	$P(F\mid P)=\dfrac{P(F\cap P)}{P(P)}$	**M1**	P(P) consistent with their tree diagram seen anywhere
	$=\dfrac{0.15\times0.65}{0.85+0.15\times0.65}$ or $\dfrac{0.15\times0.65}{1-0.15\times0.35}$	**A1**	Correct unsimplified P(P) seen as num or denom of a fraction
	$=\dfrac{0.0975}{0.9475}$	**M1**	P($F\cap P$) found as correct product or consistent with their tree diagram seen as num or denom of a fraction
	$=\dfrac{39}{379}=0.103$	**A1**	
		4	

178. 9709_w17_qp_63 Q: 6

A car park has spaces for 18 cars, arranged in a line. On one day there are 5 cars, of different makes, parked in randomly chosen positions and 13 empty spaces.

(i) Find the number of possible arrangements of the 5 cars in the car park. [2]

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(ii) Find the probability that the 5 cars are not all next to each other. [5]

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On another day, 12 cars of different makes are parked in the car park. 5 of these cars are red, 4 are white and 3 are black. Elizabeth selects 3 of these cars.

(iii) Find the number of selections Elizabeth can make that include cars of at least 2 different colours. [5]

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Answer:

Question	Answer	Marks	Guidance
(i)	$^{18}P_5$	M1	$^{18}P_x$ or $^{y}P_5$ OE seen, $0 < x < 18$ and $5 < y < 18$, can be mult by $k \geqslant 1$
	= 1 028 160	A1	
		2	

Question	Answer	Marks	Guidance
(ii)	*EITHER:* e.g. ***(CCCCC)********** in 5!×14 ways	(B1	5! OE mult by k ⩾ 1, considering the arrangements of cars next to each other
	= 1680	B1	Mult by 14 OE, (or 14 on its own) considering positions within the line
	P (next to each other) = 1680/1 028 160	M1	Dividing by **(i)** for probability
	P(not next to each other) = 1 – 1680/1 028 160	M1	Subtracting prob from 1 (or their ‘5! × 14’ from **(i)**)
	$= 0.998 \left(\frac{611}{612}\right)$ OE	A1)	
	OR1: $\frac{5! \times 14!}{18!} = 0.001634$	(B1	5! OE mult by k ⩾ 1 (on its own or in numerator of fraction) considering the arrangements of cars next to each other
		B1	Multiply by 14!, (or 14! on its own) considering all ways of arranging spaces with 5 cars together
		M1	Dividing by 18!, total number of ways of arranging spaces
	1 – 0.001634	M1	Subtracting prob from 1 (or ‘5! × 14!’ from 18!)
	= 0.998(366)	A1)	
	OR2: 4 together – $2\times5!\times14C12 = 21\,840$ 3, 1, 1 – $3\times5!\times14C11 = 131\,040$ 3, 2 – $2\times5!\times14C12 = 21\,840$ 2,2,1 – $3\times5!\times14C11 = 131\,040$ 2,1,1,1 – $4\times5!\times14C10 = 480\,480$ 1,1,1,1,1 – $5!\times14C9\,or\,14P5 = 240\,240$	(M1	Listing the six correct scenarios (only): 4 together; 3 together and 2 separate; 3 together and 2 together; two sets of 2 together and 1 separate; 2 together and 3 separate; 5 separate.
		M1	Summing total of the six scenarios, at least 2 correct unsimplified

Question	Answer	Marks	Guidance
	Total = 1 026 480	A1	Total of 1 026 480
		M1	Dividing their 1 026 480 by their **6(i)**
	$1\,026\,480 \div 1\,028\,160 = 0.998(366)$	A1)	
		5	

Question	Answer	Marks	Guidance
(iii)	R(5) W(4) B(3) Scenarios No. of ways 1 1 1 = 5 × 4 × 3 = 60 0 1 2 = 4 × $^3C_2$ = 12 0 2 1 = $^4C_2$ × 3 = 18 1 0 2 = 5 × $^3C_2$ = 15 2 0 1 = $^5C_2$ × 3 = 30 1 2 0 = 5 × $^4C_2$ = 30 2 1 0 = $^5C_2$ × 4 = 40	B1	$5C1 \times 4C1 \times 3C1$ or better seen i.e. no. of ways with 3 different colours
		M1	Any of $^5C_2$ or $^4C_2$ or $^3C_2$ seen multiplied by $k > 1$ (can be implied)
		A1	2 correct unsimplified 'no. of ways' other than 5C1 × 4C1 × 3C1
		M1	Summing no more than 7 scenario totals containing at least 6 correct scenarios
	Total = 205	A1	
	OR		
	$^{12}C_3$ –	M1	Seeing '$^{12}C_3$ –', considering all selections of 3 cars
	– $^5C_3$	M1	Subt $^5C_3$ OE, removing only red selections
	– $^4C_3$	M1	Subt $^4C_3$ OE, removing only white selections
	– $^3C_3$	M1	Subt $^3C_3$ OE, removing only black selections
	= 205	A1	Correct answer
		5	

179. 9709_m16_qp_62 Q: 3

A fair eight-sided die has faces marked 1, 2, 3, 4, 5, 6, 7, 8. The score when the die is thrown is the number on the face the die lands on. The die is thrown twice.

- Event R is 'one of the scores is exactly 3 greater than the other score'.
- Event S is 'the product of the scores is more than 19'.

(i) Find the probability of R. [2]

(ii) Find the probability of S. [2]

(iii) Determine whether events R and S are independent. Justify your answer. [3]

Answer:

(i)	P(R) [(1, 4),(2,5),(3,6),(4,7),(5,8)] × 2/64 = 10/64	**M1** **A1** 2	List of at least 4 different options or possibility space diagram Correct answer
(ii)	P(S) = [(3,8)(3,7)(4,8)(4,7)(4,6)(4,5)(5,8)(5,7)(5,6)(6,8)(6,7)(7,8)] × 2 + (5,5)(6,6)(7,7)(8,8) = 28/64	**M1** **A1** 2	List of at least 14 different options or ticks **oe** from possibility space Correct answer
(iii)	$P(R \cap S) = 4/64$ 4/64 ≠ 10/64 × 28/64 Events are not independent	**B1** **M1** **A1** 3	Comparing their P(R∩S) with (i) ×(ii) with values Correct answer

180. 9709_m16_qp_62 Q: 5

In a certain town, 35% of the people take a holiday abroad and 65% take a holiday in their own country. Of those going abroad 80% go to the seaside, 15% go camping and 5% take a city break. Of those taking a holiday in their own country, 20% go to the seaside and the rest are divided equally between camping and a city break.

(i) A person is chosen at random. Given that the person chosen goes camping, find the probability that the person goes abroad. [5]

(ii) A group of n people is chosen randomly. The probability of all the people in the group taking a holiday in their own country is less than 0.002. Find the smallest possible value of n. [3]

Answer:

(i)	P(Abroad given camping)	**M1**		Attempt at P($A \cap C$) seen alone anywhere
	$= \dfrac{P(A \cap C)}{P(A \cap C) + P(H \cap C)}$	**A1**		Correct answer seen as num or denom of a fraction
	$= \dfrac{0.35 \times 0.15}{0.35 \times 0.15 + 0.65 \times 0.4}$	**M1**		Attempt at P(C) seen anywhere
	$= \dfrac{0.0525}{0.3125}$	**A1**		Correct unsimplified answer seen as num or denom of a fraction
	$= 0.168$	**A1**	5	Correct answer
(ii)	$(0.65)^n < 0.002$	**M1**		Eqn with 0.65 or 0.35, power n, 0.002 or 0.998
	$n > \lg(0.002)/\lg(0.65)$	**M1**		Attempt to solve their eqn by logs or trial and error need a power
	$n = 15$	**A1**	3	Correct answer

181. 9709_s16_qp_61 Q: 3

The probability that the school bus is on time on any particular day is 0.6. If the bus is on time the probability that Sam the driver gets a cup of coffee is 0.9. If the bus is not on time the probability that Sam gets a cup of coffee is 0.3.

(i) Find the probability that Sam gets a cup of coffee. [2]

(ii) Given that Sam does not get a cup of coffee, find the probability that the bus is not on time. [3]

Answer:

(i)	P (cup of coffee) = 0.6×0.9 + 0.4× 0.3	**M1**		Summing two 2-factor probabilities
	= 0.66	**A1**	[2]	Correct answer accept 0.660
(ii)	P(Not on time | no cup of coffee)	**M1**		0.4×0.7 seen as num or denom of a fraction
	$= \dfrac{P(\text{not on time} \cap \text{no cup})}{P(\text{no cup})} = \dfrac{0.4 \times 0.7}{1 - 0.66}$	**M1**		Attempt at P(no cup) as $0.1 \times p_1 + 0.7 \times p_2$ or as 1 – (i) seen anywhere
	$= \dfrac{0.28}{0.34} = 0.824$	**A1**	[3]	

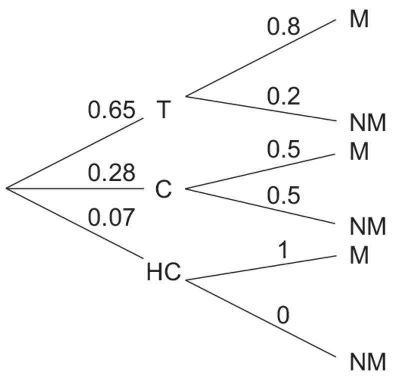
182. 9709_s16_qp_62 Q: 1

Ayman's breakfast drink is tea, coffee or hot chocolate with probabilities 0.65, 0.28, 0.07 respectively. When he drinks tea, the probability that he has milk in it is 0.8. When he drinks coffee, the probability that he has milk in it is 0.5. When he drinks hot chocolate he always has milk in it.

(i) Draw a fully labelled tree diagram to represent this information. [2]

(ii) Find the probability that Ayman's breakfast drink is coffee, given that his drink has milk in it. [3]

Answer:

Qu	Answer	Marks	Notes
(i)		M1	Correct shape with either one branch after HC or 2 branches with 0 prob seen correct Labelled and clear annotation
		A1 [2]	All probs correct
(ii)	$P(C \mid \text{milk}) = \frac{P(coffee \cap milk)}{P(milk)}$	M1	Attempt at P(coffee∩ milk)as a two-factor prod only seen as num or denom of a fraction
	$= \frac{0.28 \times 0.5}{0.65 \times 0.8 + 0.28 \times 0.5 + 0.07(\times 1)}$	M1	Summing appropriate three 2-factor products seen anywhere (can omit the 1)
	$= \frac{0.14}{0.73}$		
	=0.192	A1 [3]	Correct answer oe

183. 9709_s16_qp_63 Q: 1

In a group of 30 adults, 25 are right-handed and 8 wear spectacles. The number who are right-handed and do not wear spectacles is 19.

(i) Copy and complete the following table to show the number of adults in each category. [2]

	Wears spectacles	Does not wear spectacles	Total
Right-handed			
Not right-handed			
Total			30

An adult is chosen at random from the group. Event X is 'the adult chosen is right-handed'; event Y is 'the adult chosen wears spectacles'.

(ii) Determine whether X and Y are independent events, justifying your answer. [3]

Answer:

Qu	Answer	Marks	Guidance
(i)	<table><tr><td></td><td>Wears specs</td><td>Not wears specs</td><td>Total</td></tr><tr><td>RH</td><td>6</td><td>19</td><td>25</td></tr><tr><td>Not RH</td><td>2</td><td>3</td><td>5</td></tr><tr><td>Total</td><td>8</td><td>22</td><td></td></tr></table>	B1 B1 [2]	One correct row or col including total other than the Total row/column All correct
(ii)	P(X) = 25/30, P(Y) = 8/30	M1	P(X) or P(Y) from their table or correct from question (denom 30) oe
	P(X) × P(Y) = 25/30 × 8/30 = 200/900 = 2/9 P(X∩Y) = 6/30 = 1/5 ≠ P(X) × P(Y)	M1	Comparing their P(X) × P(Y) (values substituted) with their evaluated P(X∩Y) – not P(X)×P(Y)
	Not independent	A1 [3]	

184. 9709_w16_qp_61 Q: 6

Deeti has 3 red pens and 1 blue pen in her left pocket and 3 red pens and 1 blue pen in her right pocket. 'Operation T' consists of Deeti taking one pen at random from her left pocket and placing it in her right pocket, then taking one pen at random from her right pocket and placing it in her left pocket.

(i) Find the probability that, when Deeti carries out operation T, she takes a blue pen from her left pocket and then a blue pen from her right pocket. [2]

The random variable X is the number of blue pens in Deeti's left pocket after carrying out operation T.

(ii) Find P(X = 1). [3]

(iii) Given that the pen taken from Deeti's right pocket is blue, find the probability that the pen taken from Deeti's left pocket is blue. [4]

Answer:

(i)	P(B, B) = 1/4 ×2/5 = 1/10	**M1** **A1**	 [2]	Multiplying two different probs
(ii)	P(X = 1) = P(R,R) + P(B,B) = 3/4 × 4/5 + 1/10 = 14/20 (7/10)	**M1** **M1** **A1**	 [3]	Finding P(R, R) (=3/5) Summing two options
(iii)	P(B B) $=\dfrac{P(B\cap B)}{P(B)}=\dfrac{1/10}{3/4\times1/5+1/4\times2/5}$ = 2/5	**M1** **M1** **A1** **A1**	 [4]	their (i) seen as num or denom of a fraction ¾ × p_1 + ¼ × p_2 seen anywhere 1/4 (unsimplified) seen as num or denom of a fraction, www

185. 9709_w16_qp_62 Q: 1

When Anya goes to school, the probability that she walks is 0.3 and the probability that she cycles is 0.65; if she does not walk or cycle she takes the bus. When Anya walks the probability that she is late is 0.15. When she cycles the probability that she is late is 0.1 and when she takes the bus the probability that she is late is 0.6. Given that Anya is late, find the probability that she cycles. [5]

Answer:

	P(C given L) = $\dfrac{P(C\cap L)}{P(L)}$	**M1**		P(C∩L) seen as num or denom of a fraction
	$=\dfrac{0.65\times0.1}{0.65\times0.1+0.3\times0.15+0.05\times0.6}$	**A1**		Correct unsimplified P(C∩L) as numerator
		M1		Summing three 2-factor products seen anywhere
	$=\dfrac{0.065}{0.14}$	**A1**		0.14 (unsimplified) seen as num or denom of a fraction
	$= 0.464,\ \dfrac{13}{28}$	**A1**	[5]	oe

186. 9709_w16_qp_63 Q: 4

For a group of 250 cars the numbers, classified by colour and country of manufacture, are shown in the table.

	Germany	Japan	Korea
Silver	40	26	34
White	32	22	26
Red	28	12	30

One car is selected at random from this group. Find the probability that the selected car is

(i) a red or silver car manufactured in Korea, [1]

(ii) not manufactured in Japan. [1]

X is the event that the selected car is white. Y is the event that the selected car is manufactured in Germany.

(iii) By using appropriate probabilities, determine whether events X and Y are independent. [5]

Answer:

(i)	64/250, 0.256	**B1**	[1]	oe
(ii)	190/250, 0.76(0)	**B1**	[1]	oe
(iii)	$P(X) = 80/250 = 8/25$	**M1**		attempt at $P(X)$
	$P(Y) = 100/250 = 2/5$	**M1**		attempt at $P(Y)$
	$P(X \cap Y) = 32/250 = 16/125$	**B1**		oe
	$P(X) \times P(Y) = \frac{8}{25} \times \frac{2}{5} = \frac{16}{125}$	**M1**		comparing $P(X) \times P(Y)$ and $P(X \cap Y)$ so long as independence has not been assumed
	Since $P(X) \times P(Y) = P(X \cap Y)$ therefore independent	**A1**	[5]	correct answer with all working correct

187. 9709_s15_qp_61 Q: 3

Jason throws two fair dice, each with faces numbered 1 to 6. Event A is 'one of the numbers obtained is divisible by 3 and the other number is not divisible by 3'. Event B is 'the product of the two numbers obtained is even'.

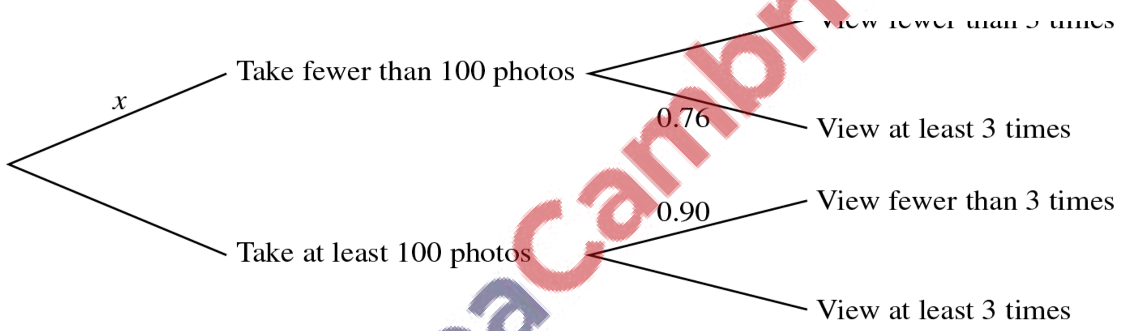
(i) Determine whether events A and B are independent, showing your working. [5]

(ii) Are events A and B mutually exclusive? Justify your answer. [1]

Answer:

(i)	$P(A) = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$ $P(B) = \frac{27}{36} = \frac{3}{4}$ $P(A \cap B) = \frac{12}{36} = \frac{1}{3}$ $P(A) \times P(B) = \frac{4}{9} \times \frac{3}{4} = \frac{1}{3}$ Independent as $P(A \cap B) = P(A) \times P(B)$	**M1** **M1** **B1** **M1** **A1 [5]**	Sensible attempt at P(*A*) Sensible attempt at P(*B*) correct $P(A \cap B)$ Cf $P(A \cap B)$ with $P(A) \times P(B)$ need at least 1 correct Correct conclusion following all correct working
(ii)	Not mutually exclusive because $P(A \cap B) \neq 0$ Or give counter example e.g. 1 and 6	**B1√ [1]**	ft their $P(A \cap B)$

188. 9709_s15_qp_61 Q: 4



A survey is undertaken to investigate how many photos people take on a one-week holiday and also how many times they view past photos. For a randomly chosen person, the probability of taking fewer than 100 photos is x. The probability that these people view past photos at least 3 times is 0.76. For those who take at least 100 photos, the probability that they view past photos fewer than 3 times is 0.90. This information is shown in the tree diagram. The probability that a randomly chosen person views past photos fewer than 3 times is 0.801.

(i) Find x. [3]

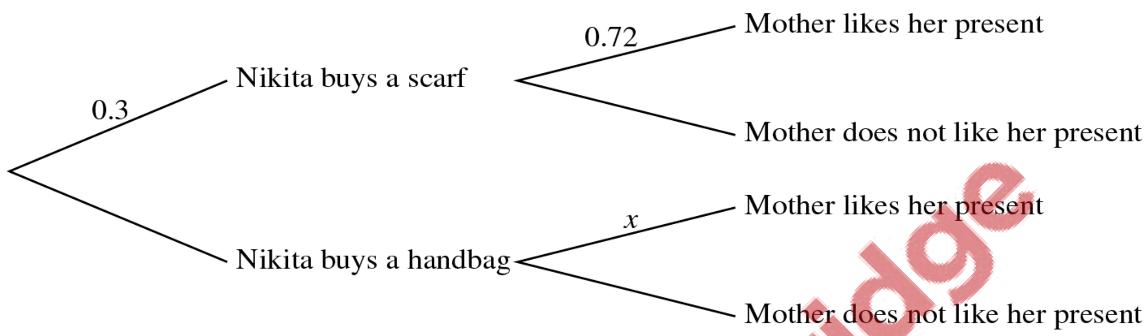
(ii) Given that a person views past photos at least 3 times, find the probability that this person takes at least 100 photos. [4]

Answer:

(i)	$(1 - x)0.9 + x \times 0.24 = 0.801$ $x = 0.15$	**M1** **A1** **A1 [3]**	Eqn with sum of two 2-factor probs = 0.801 Correct equation Correct answer

(ii)	P(≥100 times given ≤ 3 views) $\frac{P(\geqslant 100 \text{ times} \cap \geqslant 3 \text{ views})}{P(\geqslant 3 \text{ views})} =$ $\frac{0.85 \times 0.1}{0.85 \times 0.1 + 0.15 \times 0.76 \text{ or } 1 - 0.801}$ $= 0.427$	B1 M1 A1 A1 [4]	0.85×0.1 seen on its own as num or denom of a fraction Attempt at P(≥ 3 views) either ($0.85 \times p_1 + 0.15 \times p_2$) or 1 – 0.801 seen anywhere Correct unsimplified P(≥ 3 views) as num or denom of a fraction Correct answer

189. 9709_s15_qp_62 Q: 4



Nikita goes shopping to buy a birthday present for her mother. She buys either a scarf, with probability 0.3, or a handbag. The probability that her mother will like the choice of scarf is 0.72. The probability that her mother will like the choice of handbag is x. This information is shown on the tree diagram. The probability that Nikita's mother likes the present that Nikita buys is 0.783.

(i) Find x. [3]

(ii) Given that Nikita's mother does not like her present, find the probability that the present is a scarf. [4]

Answer:

(i)	$0.3 \times 0.72 + 0.7 \times x = 0.783$ $x = 0.81$	M1 A1 A1 3	Eqn with sum of two 2-factor probs =0.783 Correct equation Correct answer
(ii)	P(S given not like) $= \frac{P(S \cap NL)}{P(NL)}$ $= \frac{0.3 \times 0.28}{0.3 \times 0.28 + 0.7 \times 0.19 \text{ or } 1 - 0.783}$ $= 0.387$ (12/31)	B1 M1 A1 A1 4	0.3×0.28 seen on its own as num or denom of a fraction Attempt at P(NL) either (0.3 × p_1) + (0.7 × p_2) or 1 – 0.783 seen anywhere Correct unsimplified P(NL) as num or denom of a fraction Correct answer

190. 9709_w15_qp_62 Q: 2

A committee of 6 people is to be chosen at random from 7 men and 9 women. Find the probability that there are no men on the committee. [3]

Answer:

	P(no men) $\frac{^9C_6}{^{16}C_6} = \frac{84}{8008} = \frac{21}{2002} = \frac{3}{286}$	**B1**		$^9C_6$ seen anywhere
	$= 0.0105$	**B1**		$^{16}C_6$ seen as denom of fraction oe
		B1	3	Correct final answer
	OR $\frac{9}{16} \times \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} \times \frac{5}{12} \times \frac{4}{11} = 0.0105$	**B1**		
		B1		$(9 \times 8 \times 7 \times 6 \times 5 \times 4)$ seen anywhere
		B1		Correct unsimplified denom
				Correct final answer

191. 9709_w15_qp_63 Q: 2

In country X, 25% of people have fair hair. In country Y, 60% of people have fair hair. There are 20 million people in country X and 8 million people in country Y. A person is chosen at random from these 28 million people.

(i) Find the probability that the person chosen is from country X. [1]

(ii) Find the probability that the person chosen has fair hair. [2]

(iii) Find the probability that the person chosen is from country X, given that the person has fair hair. [2]

Answer:

(i)	$\mathrm{P}(X) = \frac{20}{28}\left(\frac{5}{7}\right)(0.714), 71.4\%$	**B1**	1	oe
(ii)	$\mathrm{P}(F) = \frac{20}{28} \times \frac{1}{4} \times \frac{8}{28} \times \frac{6}{10} = \frac{7}{20}$	**M1**		Summing two 2-factor probs created by One of ¼ or ¾ multiplied by 20/28 or 8/28 Added to 4/10 or 6/10 × altn population prob
		A1	2	Correct answer
(iii)	$\mathrm{P}(X \mid F) = \frac{5/28}{7/20} = \frac{25}{49}(0.510)$	**M1**		Their unsimplified country X probability (5/28) as num or denom of a fraction Or (their fair hair population) ÷ (total fair hair pop)
		A1	2	Correct answer

192. 9709_w15_qp_63 Q: 3

Ellie throws two fair tetrahedral dice, each with faces numbered 1, 2, 3 and 4. She notes the numbers on the faces that the dice land on. Event S is 'the sum of the two numbers is 4'. Event T is 'the product of the two numbers is an odd number'.

(i) Determine whether events S and T are independent, showing your working. [5]

(ii) Are events S and T exclusive? Justify your answer. [1]

Answer:

(i)	$P(S) = \frac{3}{16}$	M1		Sensible attempt at P(*S*)
	$P(T) = \frac{4}{16}$	M1		Sensible attempt at P(*T*)
	$P(S \cap T) = \frac{2}{16}$	B1		Correct $P(S \cap T)$
	$P(S) \times P(T) = \frac{3}{64} \neq \frac{2}{16}$	M1		comp $P(S) \times P(T)$ with $P(S \cap T)$ (their values), evaluated
	Not independent	A1	5	Correct conclusion following all correct working
(ii)	not exclusive since $P(S \cap T) \neq 0$ Or counter example e.g. 1 and 3 Or $P(S \cup T) \neq P(S)+P(T)$ with values	B1√	1	FT their $P(S \cap T)$, not obtained from $P(S) \times P(T)$, with value and statement.